Fracture Mechanics – Workshop Solutions

1. Given: W = 50 mm., B = 25 mm, a_o = 25 mm, P = 25 kN

a) a/W = 25/50 = 0.5, then f = 9.66

from compact specimen K calibration table in Lecture 2.1

K = Pf/B√W = (.025 MN)(9.66)/(.025 m)(√.05 m) = 43.2 MPa√m

Let Δa = 5 mm then a = 30, a/W = 0.6, f = 13.65,

K = (5)(13.65)/(1)(√2) = 61.0

b) Similar calibrations a = 250, W = 500, P = 80, K = 43.7
a = 255, a/W = .51, f = 9.96, K = 45.1

c) CCT use 2.1 from Tada Handbook (in Section 2)

a = 25, b(W) = 5.0 = 2W/2, a/b = 0.2, F = 1.0246;

σ = 1.0 MN/(.025x.25) m^2 = 160 MPa

$$K = \sigma\sqrt{\pi a}F$$

K = (160)√(.025π) (1.0246) = 45.9 MPa√m

Δa = 5, a = 30, a/b = 0.24, F = 1.032, K = 50.7

Alternate form: for a/W = 0.2, f = 0.406, P = 1000, K = (1.0)(.406)/.025√.125 = 45.9

2. K same form as 1c. from Tada 2.10, σ = 200 MPa, b = 100 mm

$$K = \sigma\sqrt{\pi a}F$$

$F = \text{graphical value}/(1 - a/b)^{3/2}/(1 - a/b)^{3/2}$

Example: $a = 2.5$, $a/b = .025$, graph $= 1.1$, $F = 1.1/\ (1 - 2.5/100)^{3/2} = 1.14$

$K = 200\sqrt{\ }(.0025\pi)1.14 = 20.2$

a	a/b	graphical value	$(1\text{-}a/b)^{3/2}$	F	K
2.5	.025	1.1	.963	1.14	20.2
12.5	.125	1.0	.818	1.23	48.7
37.5	.375	.98	.494	1.98	135.9

3. $r_y = (1/2\pi)(K/\sigma_{ys})^2$

$K = 43.2$ MPa$\sqrt{m}$, $\sigma_{ys} = 240$ MPa, $r_y = (1/2\pi)(43.2/240)^2 = .0052$ m $= 5.2$ mm

W - a = 25 mm, $r_y/(W - a) = 0.208 = 21\%$

$\sigma_{ys} = 1200$, $r_y = 0..21$, 0.8%

after $\Delta a = 0.2$, $K = 61$, $r_y = (1/2\pi)(61/240)^2 = 0.0103$m $= 10.3$ mm

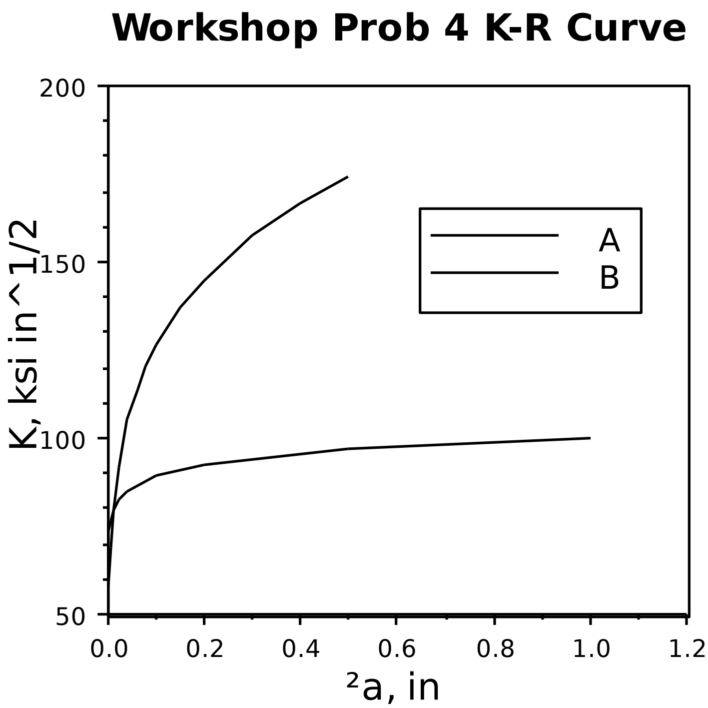
$\sigma_{ys} = 240$ MPa , W - a = 20, $r_y/(W\text{-}a) = 0.515$ or 52 %

$\sigma_{ys} = 1200$, $r_y = 0.41$ mm, 2.0%

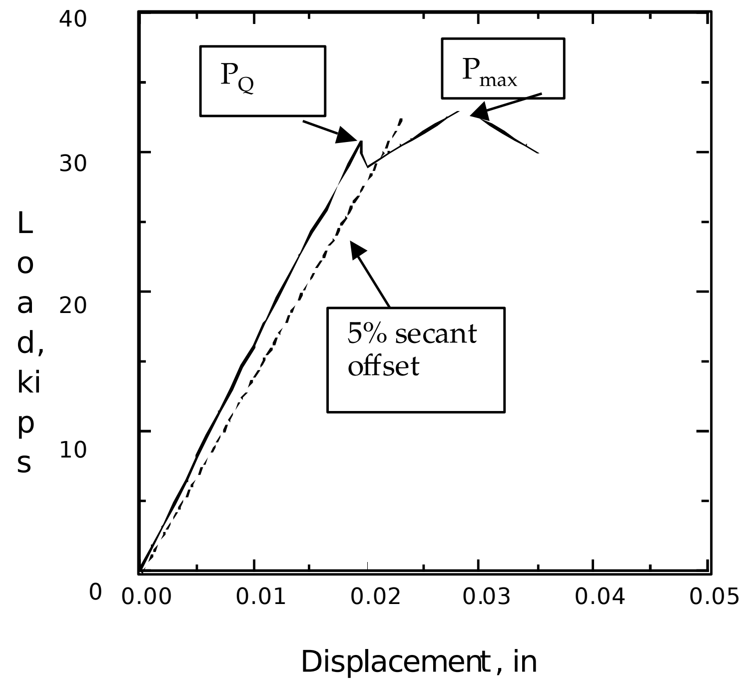
4. $K_1 = 200(\Delta a)^{0.2}$, $K_2 = 100\ (\Delta a)^{0.05}$

Δa	K_1	K_2
.01	79.6 ksi$\sqrt{in}$	79.4
.05	109.9	86.1
.1	126.2	89.1
.5	174	96.6

See plots for complete curves



5. Approximate reading of P - v graph



$P_Q = 31$ kips, $P_{max} = 33$

$a/W = 2.1/4 = 0.525$, $f = 10.45$

$K_Q = (31)(10.45)/2\sqrt{4} = 81$ ksi$\sqrt{\text{in}}$,

Validity: $P_{max}/ P_Q = 33/31 = 1.065 < 1.1$, OK

$2.5(K_Q/ \sigma_{ys})^2 = 2.5(81/100)^2 = 1.64 < 2$ (B) or 2.1(a) OK

$0.45 < 0.525 < 0.55$, a_o/W OK

$K_Q = K_{Ic} = 81$ ksi$\sqrt{\text{in}}$

$R_{sc} = [2P_{max}(2W + a)]/[B(W - a)^2\sigma_{ys}]$

$= [2\ (33\text{ kips})(2\text{x}4 + 2.1)\text{ in}]/[(2.0\text{in})(1.9\text{ in})^2(100\text{ ksi})] = 0.92$

(in the linear elastic range)

K_{Ic} SI:

$P_Q = 31$ kipsx(4.448) = 138 kN, $P_{max} =$ 148 kN

$a/W = 52.5/100 = 0.525$, $f = 10.45$

$K_Q = (0.138)(10.45)/.05\sqrt{.1} = 91.2$ MPa$\sqrt{m}$,

Validity: $P_{max}/ P_Q = 148/138 = 1.07 < 1.1$, OK

$2.5(K_Q/ \sigma_{ys})^2 = 2.5(91.2/700)^2 = 0.042$ m = 42.4 mm < 50 (B) or 52.2(a) OK

$0.45 < 0.525 < 0.55$, a_o/W OK

$K_Q = K_{Ic} = 91.2$ MPa$\sqrt{m}$

$R_{sc} = [2P_{max}(2W + a)]/[B(W - a)^2\sigma_{ys}]$

$= [2\ (.148)(2x.1 + .052)$ in$]/[(.05)(.0475)^2(700)] = 0.94$

(in the linear elastic range)

6. Take points 5 and 6 as example

$da/dN = (0.79 - 0.73)/(125{,}000 - 95{,}000) = 0.060/30{,}000 = 2.0\text{x}10^{-6}$

$a_{ave} = (0.73 + 0.79)/2 = 0.76$, $a/W = 0.38$, $f = 6.91$, $\Delta P = 1000 - 100 = 900$,

$\Delta K = (900)(6.91)/(0.25\sqrt{2}) = 17{,}600\ \text{psi}\sqrt{\text{in}} = 17.6\ \text{ksi}\sqrt{\text{in}}$

da/dN, in/cyc	ΔK, ksi√in
$1.162\text{x}10^{-6}$	14.5
$1.351\text{x}10^{-6}$	15.0
$1.333\ \text{x}10^{-6}$	1.5.7
$1.6\ \text{x}10^{-6}$	16.5
$2.0\ \text{x}10^{-6}$	17.6
$2.33\ \text{x}10^{-6}$	19.1
$3.0\ \text{x}10^{-6}$e	21.4
$4.4\ \text{x}10^{-6}$	25.2
$8.0\ \text{x}10^{-6}$	30.0
$16.0\ \text{x}10^{-6}$	39.3
$40\ \text{x}10^{-6}$	61.3

Plot log da/dN vs ΔK and fit equation result is,

$da/dN = 1.44\text{x}10^{-9}\ (\Delta K)^{2.5}$

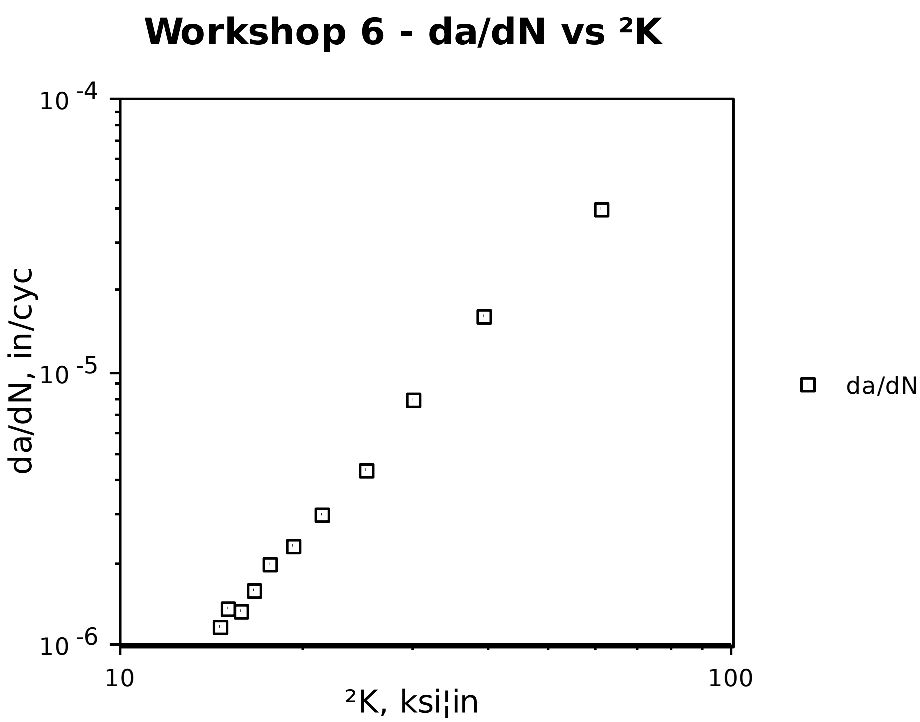
(see attached plots)

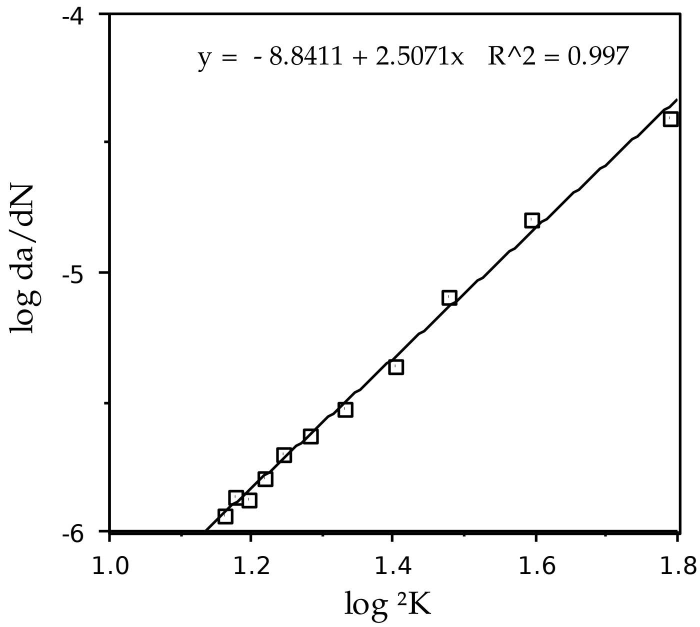
7. $\Delta K = 5.0$ ksi$\sqrt{in}$, $a = 0.6$ in, $a/W = 0.3$, $f = 5.62$

$\Delta K = \Delta P f / B\sqrt{W} = \Delta P(5.62)/(0.25\sqrt{2}) = 5$,

$\Delta P = 0.315$ kips

or P ranges from 100 to 415 lbs





Workshop Prob 6 log - log fit

8. $W = 2.0$, $B = 1.0$, $a_o = 1.0$ (all in.) $a/W = 0.5$, $E = 30{,}000$ ksi

$BEv/P = 36.98$, $f = 9.66$

$P = (1.0)(30{,}000)(0.012)/36.98 = 9.73$ kips

$K_o = (9.73)(9.66)/(1\sqrt{2}) = 66.5$

$a_f = 1.5$, $a/W = 0.75$, $BEv/P = 185.4$, $f = 28.86$,

$P = (1.0)(30{,}000)(0.012)/185.4 = 1.94$ kips

$K_f = (1.94)(28.86)/(1\sqrt{2}) = 39.6$ ksi$\sqrt{\text{in}}$

Workshop 4

1. $W = 0.05$ m, $B = 0.025$ m, $a_o = 0.04$ m $a/W = 0.5$, $E = 210{,}000$ MPa

$BEv/P = 22.88$, $f = 7.28$

$P = (0.025)(210{,}000)(0.00025)/22.88 = 0.05736$ MN (57.4kN)

$K_o = (0.05736)(7.28)/(0.025\sqrt{0.05}) = 74.7$ MPa$\sqrt{m}$

$a_f = 40$, $a/W = 0.80$, $BEv/P = 305.5$, $f = 41.2$,

$P = (0.025)(210{,}000)(0.00025)/305.5 = 0.0043$ MN

$K_f = (0.0043)(41.2)/(0.025\sqrt{0.05}) = 31.7$ MPa$\sqrt{m}$

2. $\sigma = P/A = 1.0\ /(0.025x0.1) = 400$ MPa, $K = \sigma\sqrt{(\pi a)}\ F$,

a) for 91.2 MPa√m from Workshop 2, cannot solve a_{cr} directly make a table

a, mm	a/b	F	K, MPa√m	comment
Try 20	0.2	1.37	137.4	too high
10	0.1	1.195	84.7	too low
11	0.11	1.21	90.0	close
11.2	0.112	1.215	91.2	good

Then, $a_{cr} = 11.2$ mm

Alternate form: $K = 91.2 = Pf/B\sqrt{W} = 1.0f/0.025\sqrt{.1}$;
$f = 0.721$, $a/W = .1125$, $a_{cr} = 11.25$ mm

b) For $= \sigma_{ys} = 350$ and $\sigma = 400$ the plate is already yielded. No defect allowed

$\sigma = P/A = 1.0\ /(1x5) = 50$ ksi, $K = \sigma\sqrt{(\pi a)}\ F$,

a) for $K_{Ic} = 81.0$ from Prob. 5, cannot solve a_{cr} directly make a table

a, in	a/b	F	K, ksi√in	comment
Try 1.0	0.2	1.37	121	too high
0.5	0.1	1.21	75.8	too low
0.55	0.11	1.22	80.2	close
0.56	0.112	1.22	80.9	good

Then, $a_{cr} = 0.56$ in

Alternate form: $K = 81 = Pf/B\sqrt{W} = 250f/1\sqrt{5}$; $f = 0.724$, $a/W = .1135$, $a_{cr} = 0.57$ in

b) For $= \sigma_{ys} = 50$ and $\sigma = 50$ the plate is already yielded. No defect allowed

10. $\sigma = 50$ ksi, $K = 1.122\sigma\sqrt{(\pi a)}$

$K_{IEAC} = 40 = 1.122(50\sqrt{(\pi a)}$, $a_{EAC} = 0.162$ in.

$K_{Ic} = 100 = 1.122(50\sqrt{(\pi a)}$, $a_{cr} = 1.011$ in.

11. $da/dN = 1.44x10^{-9}\ \Delta K^{2.5}$

$a_o = 0.1$, $K_{Ic} = 100$, $\Delta\sigma = 30$, $\sigma_{max} = 40$

$a_f = (1/\pi)(100/1.122x40)^2 = 1.58$

$da/dN = 1.44x10^{-9}[1.122x30\sqrt{(\pi a)}]^{2.5} = 3.96x10^{-5}a^{1.25}$

$$N_f = \int_{0.1}^{1.58} \frac{da}{3.96x10^{-5}a^{1.25}} = \frac{1}{3.96x10^{-5}}\left[\frac{a^{-0.25}}{(-0.25)}\right]_{0.1}^{1.58}$$

N_f = 89,550 cycles

12. Treat the pressure vessel wall as a CCT

$a_{cr} = (1/\pi)(120/40)^2 = 2.86$ in

this is the through crack that will cause failure

a) Consistent shape, $a/2c = 0.25$, $a_o = 0.5$, $2c_o = 2.0$

$a_f = 2.0$, $2c_f = 8.0$, $c_f = 4.0 > 2.86$, break

b) Consistent growth, $\Delta a = 1.5$, $\Delta c = 1.5$, $c_o = 1.0$, $c_f = 2.5 < 2.86$, leak

(however, close to break)

13. Example of first point (all J calculations using initial crack size)

$a_o = 1.185$, $a_f = 1.268$, $\Delta a = 0.083$ in.

$a/W = 0.5925$, $f = 13.25$, $P_f = 9.50$, $K = (9.50)(13.26)/(1\sqrt{2}) = 89.0$ ksi$\sqrt{\text{in}}$,

$J_{el} = K^2(1 - \nu^2)/E = (89.1)^2(0.91)/(30,000) = 0.24$ kip-in/in^2

$J_{pl} = \eta(\text{area})/Bb = (2.213)(1020)/(1.0\text{x}0.815) = 2770$ in-lb/in^2 = 2.77 kip-in/in^2

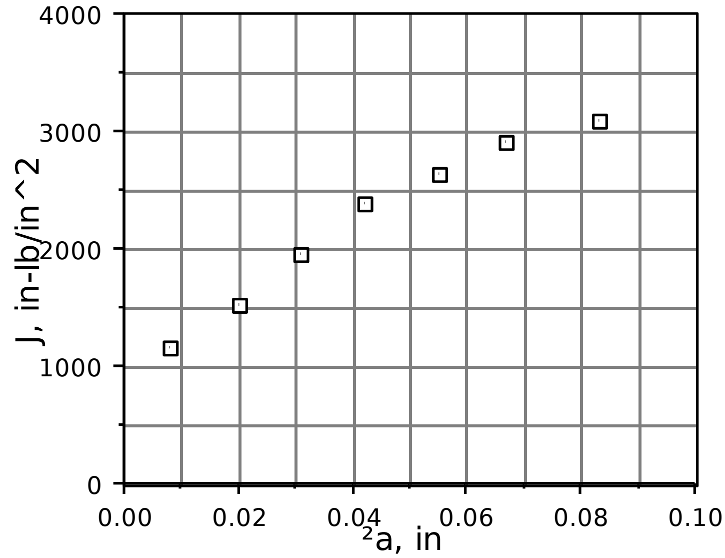
$b = W - a = 2.0 - 1.185 = 0.815$, $\eta = 2 + 0.522(b/W) = 2 + 0.522(0.815/2) = 2.213$

$J_{tot} = J_{el} + J_{pl} = 0.240 + 2.77 = 3.01$

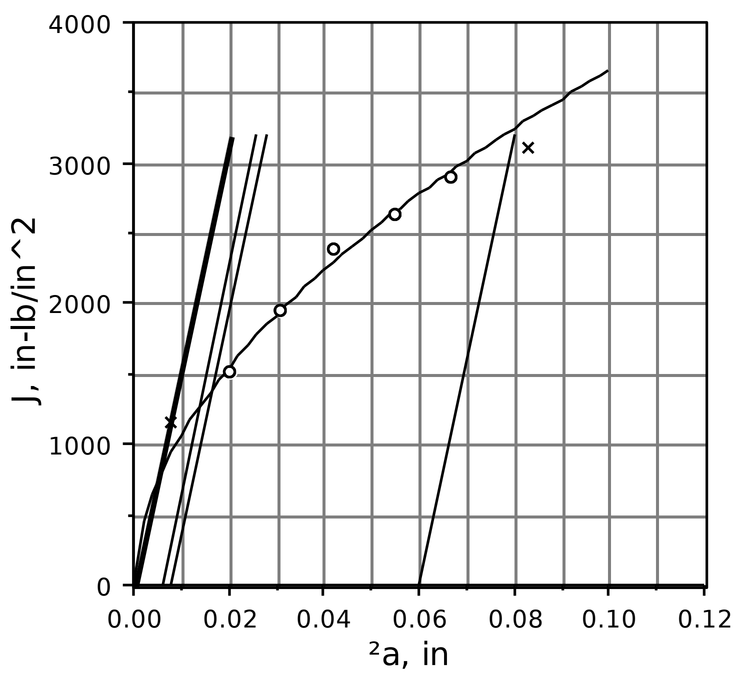
(All J values in.-lb/in^2)

J, in-lb/in2	Δa, in
3010	.083
1160	.008
2380	.042
2900	.057
1520	.020
1960	.031
2640	.055

See attached plots to do the J_{Ic} construction, $\sigma_Y = (70,000 + 90,000)/2 = 80,000$ psi



Workshop 6 - J versus ²a



Workshop 6 - J versus ²a with Construction

exclude points 1 and 2,

fit points 3 to 7 with $\ln(J) = \ln(C_1) + C_2 \ln(\Delta a)$

result $J = 12{,}597(\Delta a)^{0.537}$

solve with $J = 2 \times 80{,}000(\Delta a - 0.008)$ (J_Q calculation line)

Need table to iterate

Δa	J_{fit}	J_{line}	comment
.016	1367	1280	Too low
.017	1413	1440	Too high
.0168	1403	1408	close
.01675	1401	1400	correct

$J_Q = 1400$ in-lb/in^2

Validity

$J_{max} = b_{min}\,\sigma_Y/15 = (0.720)(80{,}000)/15 = 3840$; all points okay

$25(J_Q)/\sigma_Y = 25(1400)/(80{,}000) = 0.44$ in < all B, b

So $J_Q = J_{Ic} = 1400$ in-lb/in^2

14. $P_{max} = 12.0$, from construction, $v_{pl} = 0.0085$ in, $f = 9.66$

$K = (12)(9.66)/(1\sqrt{2}) = 82.0$

$\delta_{el} = K^2(1 - \nu^2)/(2\sigma_{ys}E) = (82)^2(.91)/[(2)(30{,}000)(60)] = 0.0017$in

$\delta_{pl} = r_p\,(W - a_o)v_{pl}/[Z + a_o + r_p\,(W - a_o)]$

load line v, $Z = 0$; CT, $r_p = 0.46$, $a_o = 1.0$

δ_{pl} = (0.0085)(0.46)(1.0)/[1 + 0.46(1.0)] = 0.00268

δ_{tot} = δ_{el} + δ_{pl} = 0.0017 + 0.00268 = 0.00438 in

15. $K_R = S_R[(8/\pi^2)) \ln\{\sec(\pi S_R/2)\}]^{-1/2}$

when $S_R = 0.9$, K_R, from above = 0.734 is the calibration point

a) $\sigma_{ys} = 150$, $\sigma = 135$, $K = \sigma\sqrt{\pi a} = 135\sqrt{(0.025\pi)} = 37.8$, $K_{Ic} = 50$

$K_R = K/K_{Ic} = 37.8/50 = 0.757 > 0.734$

so this point lies outside of the safe area, Unsafe

b) $\sigma_{ys} = 100$, $\sigma = 90$, $K = \sigma\sqrt{\pi a} = 90\sqrt{(0.025\pi)} = 25.2$, $K_{Ic} = 100$

$K_R = K/K_{Ic} = 25.2/100 = 0.252 < 0.734$

so this point lies inside of the safe area, Safe

16. $\sigma_m = 30$, $Q = 30$, $k_t = 3$ (stress concentration factor for hole in a plate)

$\sigma/\sigma_o = \varepsilon/\varepsilon_o = [k_t(\sigma_m + \sigma_b) + Q] = [(3)(30) + 30]/60 = 2.0$

From CTOD design curve, $\Phi = 1.75 = \delta_{cr}/(2\pi\, \varepsilon_o\, a)$

From prob. 14, $\delta_{cr} = 0.00493$

So, $a_{cr} = \delta_{cr}/(2\pi\, \varepsilon_o\, \Phi) = (0.00438)/[(2\pi)(60/30{,}000)(1.75)] = 0.199$ in

17. Look at points of

i) final failure with $K_{Ic} = 120$ ksi√in

ii) da/dN initiation from ΔK_{TH},

iii) Life based on da/dN vs ΔK

i) $K_{Ic} = 120$, $K = \sigma\sqrt{(\pi a)}\, F$,

$a = 1.75$, $a/b = 0.35$, $F = 1.87$

$\Delta\sigma = 0$ to $75/(1x5) = 15$ ksi

at $\sigma = 15$ ksi, $K = 15\sqrt{(1.75\pi)}(1.87) = 65.7$ to low

at $K = 120$, $\sigma = 27.4$, and $P = 137$ kips; looks like a high load

ii) Initiation flaw, $a = 0.02$, $2c = 0.1$, $a/2c = 0.2$, $Q = 1.32$,

$K = 1.122\Delta\sigma\sqrt{(\pi a/Q)}$

If $\Delta\sigma = 15$, $\Delta K = 1.122(15)\sqrt{(0.02\pi/(1.32)} = 3.7$ too low,

If $\Delta\sigma = 27.4$, $\Delta K = 6.1 > 6.0$ crack could begin to grow

Needs same high load

iii) In 7 years the devise could have
(7yr)x(365 day)x(12 hr)x(5 lifts) = 150,000 cycles

Do da/dN analysis using steel upper bound equations

Minimum life is 400,000 cycles
It would take a higher stress to shorten the life

Conclusion from this is that strut was consistently overloaded, about 140 kips instead of 75 kips the design load da/dN vs ΔK analysis

Use generic steel upper bound CGR, $a_0 = 0.02$ in , $a_f = 1.75$ in, $\Delta\sigma = 15$ ksi

$$\frac{da}{dN} = 6.6x10^{-9}\,\Delta K^{2.25}$$

$$\Delta K = 1.122\Delta\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = 6.6x10^{-9}\left[1.122(15\sqrt{\pi a}\right]^{2.25} = 1.373x10^{-5}\,a^{1.125}$$

$$N_f = \int_{0.02}^{1.75}\frac{da}{1.373x10^{-5}\,a^{1.125}} = \frac{1}{1.373x10^{-5}}\left[\frac{a^{-0.125}}{-0.125}\right]_{0.02}^{1.75} = 408{,}000$$

18. No solution here, open ended,

Look at:

i) Final crack size at beginning of dimples

ii) K_{Iscc} at beginning of intergranular surface

iii) Also try to match SS with some da/dN calculations

iv) Try to find the initiation site ΔK_{TH}