Lecture 11

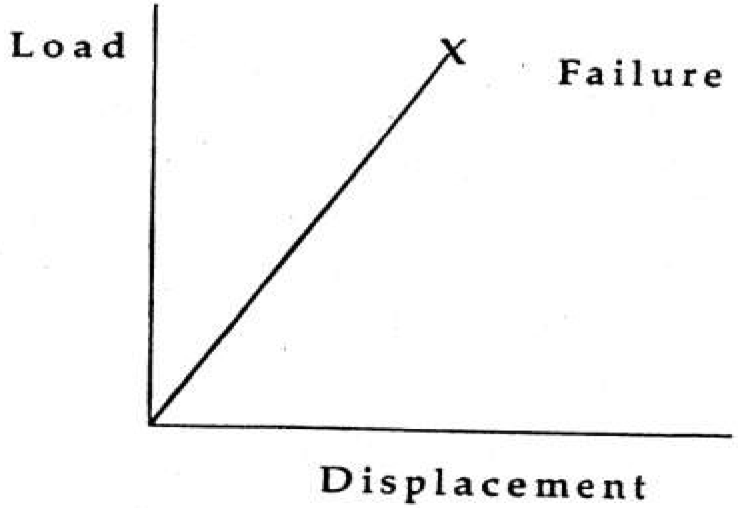
Nonlinear Fracture Mechanics Concepts

Limits of linear-elastic FM

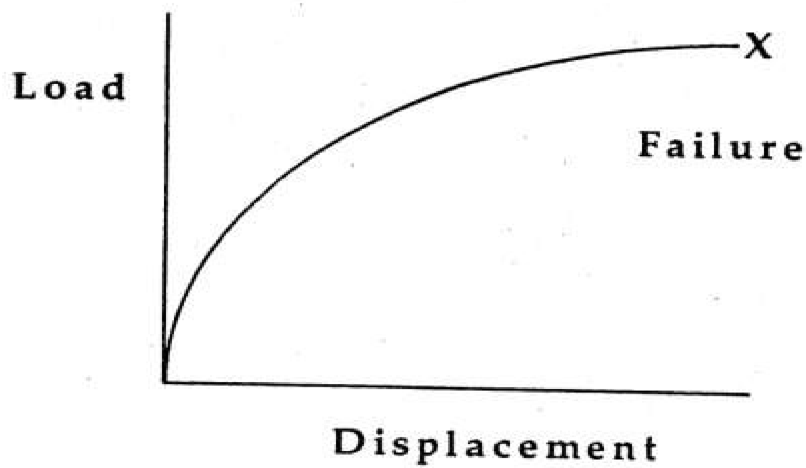
1. When large amounts of plasticity occur, K no longer characterizes the crack-tip stress field and is no longer a valid fracture parameter.

2. This limit can be determined with a plastic zone calculation or by the shape of the load versus displacement curve

3. When this happens, another approach is needed
   - An adjustment to K
   - A new parameter, J or CTOD, δ



Load versus Displacement LEFM Behavior



Load Versus Displacement EPFM Behavior

K adjustment schemes

1. Plastic zone correction

2. Dugdale strip yield model

3. Limit load

Plastic Zone Correction to K

1. Assume that the plastic zone has an effect of increasing the crack length

   $a = a_o + r_y$

   $r_y = K^2 / \alpha\pi\sigma_y{}^2$

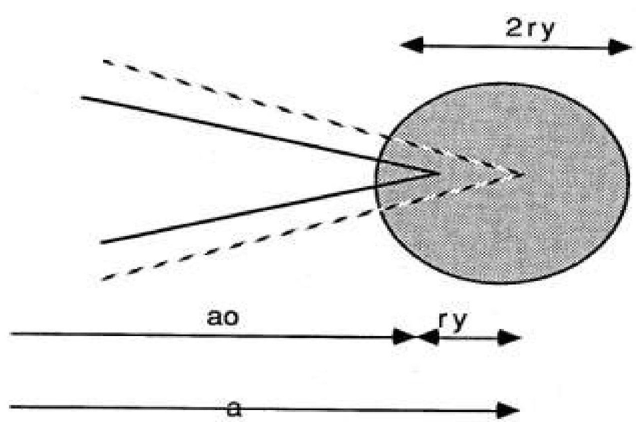
   $K = \sigma[(\pi(a_o + r_y)]^{1/2} Y(a/W)$

2. Example CCT $K = \sigma\sqrt{\pi a}$, $\alpha = 2$

   $K = \sigma[(\pi(a_o + r_y)]^{1/2} = \sigma[(\pi(a_o + K^2/2\pi\sigma_y{}^2)]^{1/2}$

then,

   $K = \sigma[(\pi a_o / \{1 - (\sigma/\sigma_y)^2/2\}]^{1/2}$



Dugdale Model

1. A strip of yielded material is assumed in front of the crack tip

2. This leads to a K solution

$$K_D = \sigma_{ys}[(8a/\pi) \{\ln \sec(\pi\sigma/2\sigma_{ys})\}]^{1/2}$$

3. The Dugdale analysis forms the basis for the R-6 diagram used for failure analysis diagram approaches

Limit Load

1 Limit load calculation approximates the point where maximum load is reached

2 It is usually based on an assumption of perfect plasticity, no strain hardening

3 Limit load is based on the Upper and Lower bound theorems in plasticity

Historical development of Elastic-Plastic Fracture mechanics, EPFM

1. CTOD, δ, Wells early 1960’s

2. J integral, late 1960’s

3. Standards
   - BSI, CTOD, 1979
   - ASTM, J_{Ic}, 1981
   - ASTM, J-R curve, 1987
   - ASTM CTOD, 1989

Definition of J

1. J is the path independent J integral of Rice

2. It is defined mathematically by

$$J = \oint_{\Gamma} W dy - \vec{T} \frac{\partial \vec{u}}{\partial x} ds$$

where

$$W = \int_{0}^{\varepsilon} \sigma d\varepsilon$$

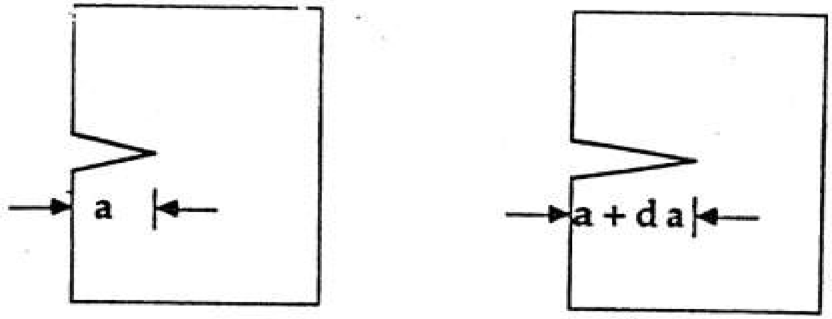
T = traction vector

u = displacement vector

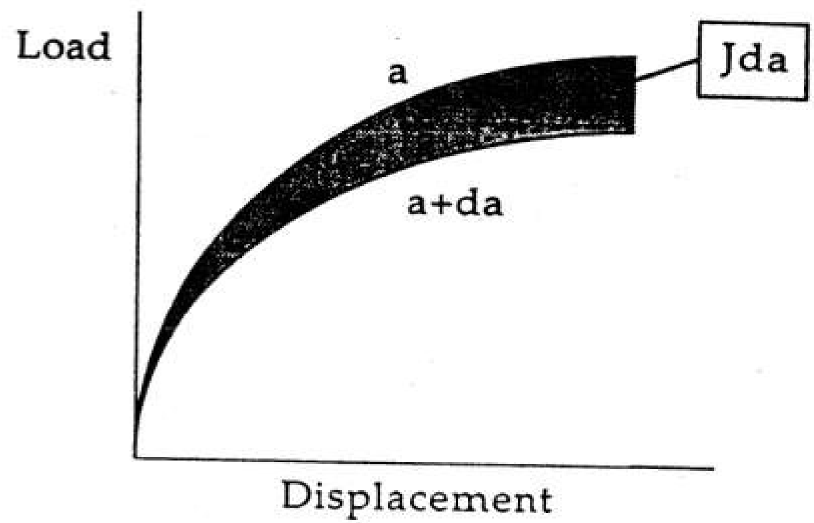
Γ is path of integration

3. This definition is not easy to use experimentally

Energy Rate Definition of J

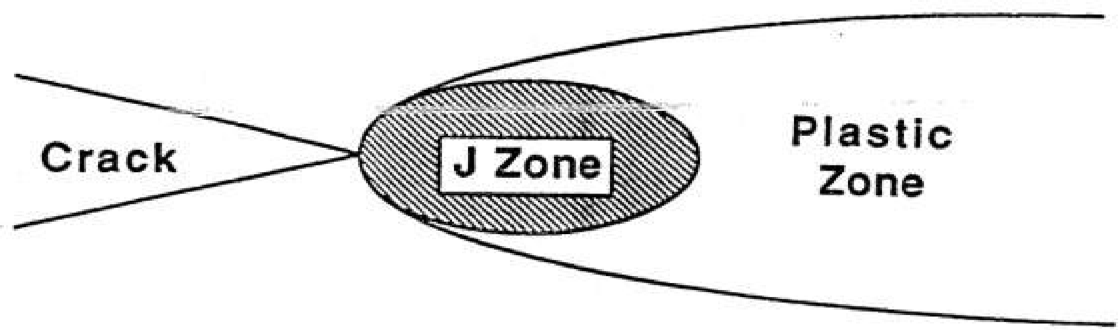


Identical Bodies, Crack Size da Difference



$$J = -\frac{1}{B}\frac{\partial U}{\partial a} = -\frac{1}{B}\frac{\partial}{\partial a}\int_0^v P\,dv$$

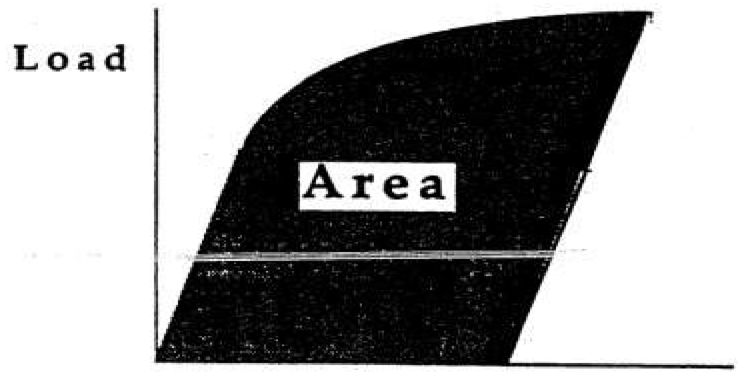
Plastic Crack Tip Field



$$\sigma_y = \left(\frac{J}{\sigma_o I_n r}\right)^{\frac{1}{n+1}} f(\theta)$$

J is the parameter which characterizes the crack tip stress field for plastic behavior. It replaces K as the fracture parameter

Basic J Formula



$$J = J_{el} + J_{pl}$$

$$= \frac{K^2(1-\nu^2)}{E} + \frac{\eta_{pl}\,\text{Area}}{B(W\text{-}a)}$$

B = thickness

W = Width

η_{pl} = Coefficient

J Solutions

1. Path integral – numerical work

2. Energy rate – experimental work

3. Handbook of J solutions

J based fracture toughness

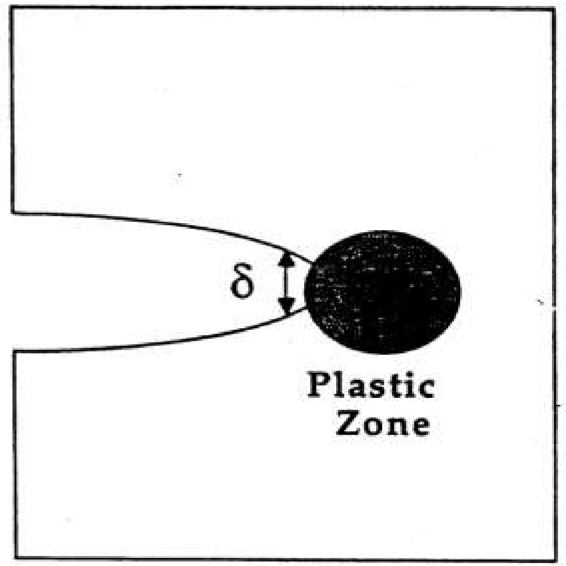
1 J_{Ic} replaces K_{Ic} as the toughness criterion for EPFM

2 J_{Ic} is a specific point on the R curve, called the J-R curve

3 It is determined by a construction procedure

4 The more general fracture toughness characterization is the J-R curve

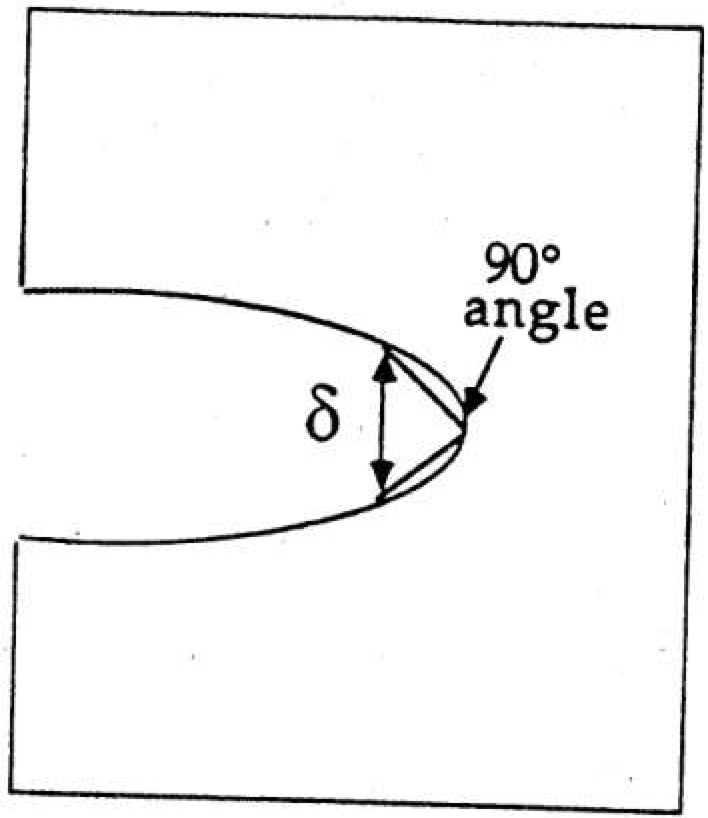
Crack Tip Opening Displacement (CTOD) or δ



1. CTOD is a crack displacement near the crack tip

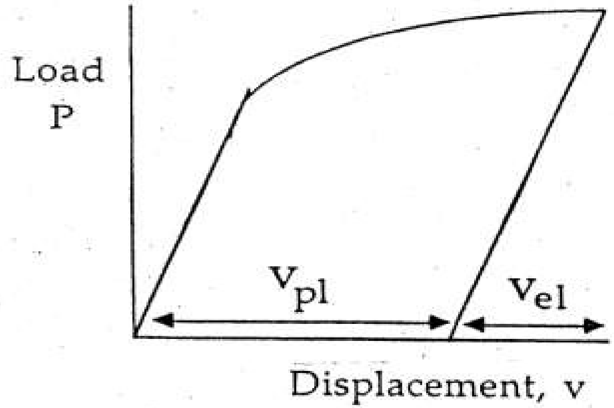
2. CTOD is related to J

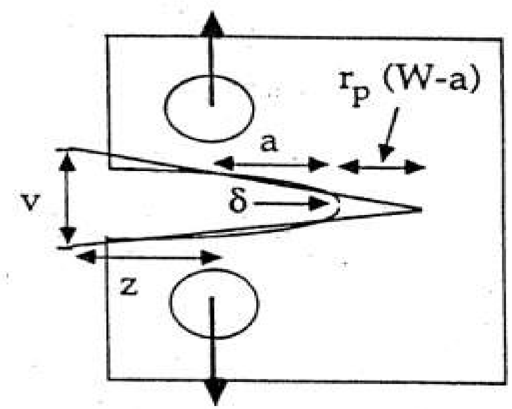
$$J = \sigma_Y \delta$$



CTOD Finite Element Definition

CTOD Equations





$$\delta = \delta_{el} + \delta_{pl}$$

$$= \frac{K^2(1-\nu^2)}{2\,\sigma_{ys}E} + \frac{r_p(W-a)v_{pl}}{r_p(W-a) + a + z}$$

Single Edge Notch Bend, SE(B), Specimen

The following equations were presented by the author at the Atlanta Task Group meeting, M 1998.

Expression for CTOD δ from the area A_p under the plot of the load P versus the plastic component of CMOD V_p, *for* $0.45 \le a/W \le 0.7$ *[for weld annex,* $0.1 \le a/W \le 0.7$*]*:

$$\delta = \frac{1}{m \cdot \sigma_Y} \cdot \left[\frac{K^2 \cdot (1 - \upsilon^2)}{E} + \frac{\eta \cdot A_p}{B \cdot (W - a) \cdot \left(1 + \frac{z}{0.8 \cdot a + 0.2 \cdot W}\right)} \right]$$

where σ_Y is the ***effective stress*** (the average of the engineering yield strength σ_{YS} and tensile strength σ_{TS}) and the ***constraint parameter m*** is a function of the aspect ratio a/ and the strain-hardening exponent n < 1, the reciprocal of the Ramberg-Osgood expon N

$$m = 1.221 + 0.793 \cdot \frac{a}{W} + 2.751 \cdot n - 1.418 \cdot \frac{a}{W} n.$$

The ***strain-hardening exponent n*** may be estimated experimentally from ASTM E 646[1] or empirically from

$$n = 1.724 + \frac{6.098}{R} + \frac{8.326}{R^2} - \frac{3.965}{R^3} \quad ($$

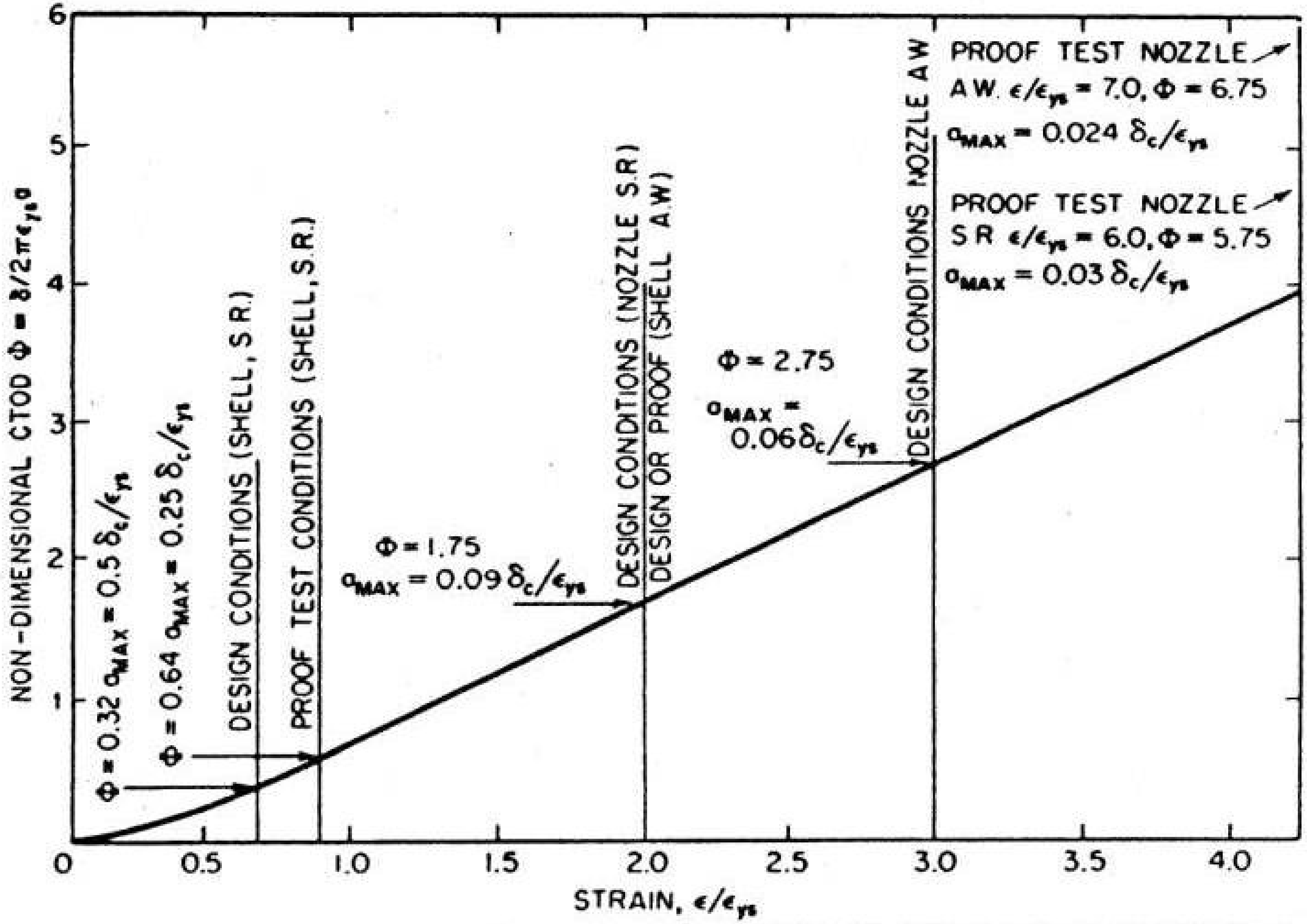
where ***R is the ratio*** of the engineering tensile strength σ_{TS} to the engineering yield strength σ_{YS}

$$R = \frac{\sigma_{TS}}{\sigma_{YS}}. \quad (4$$

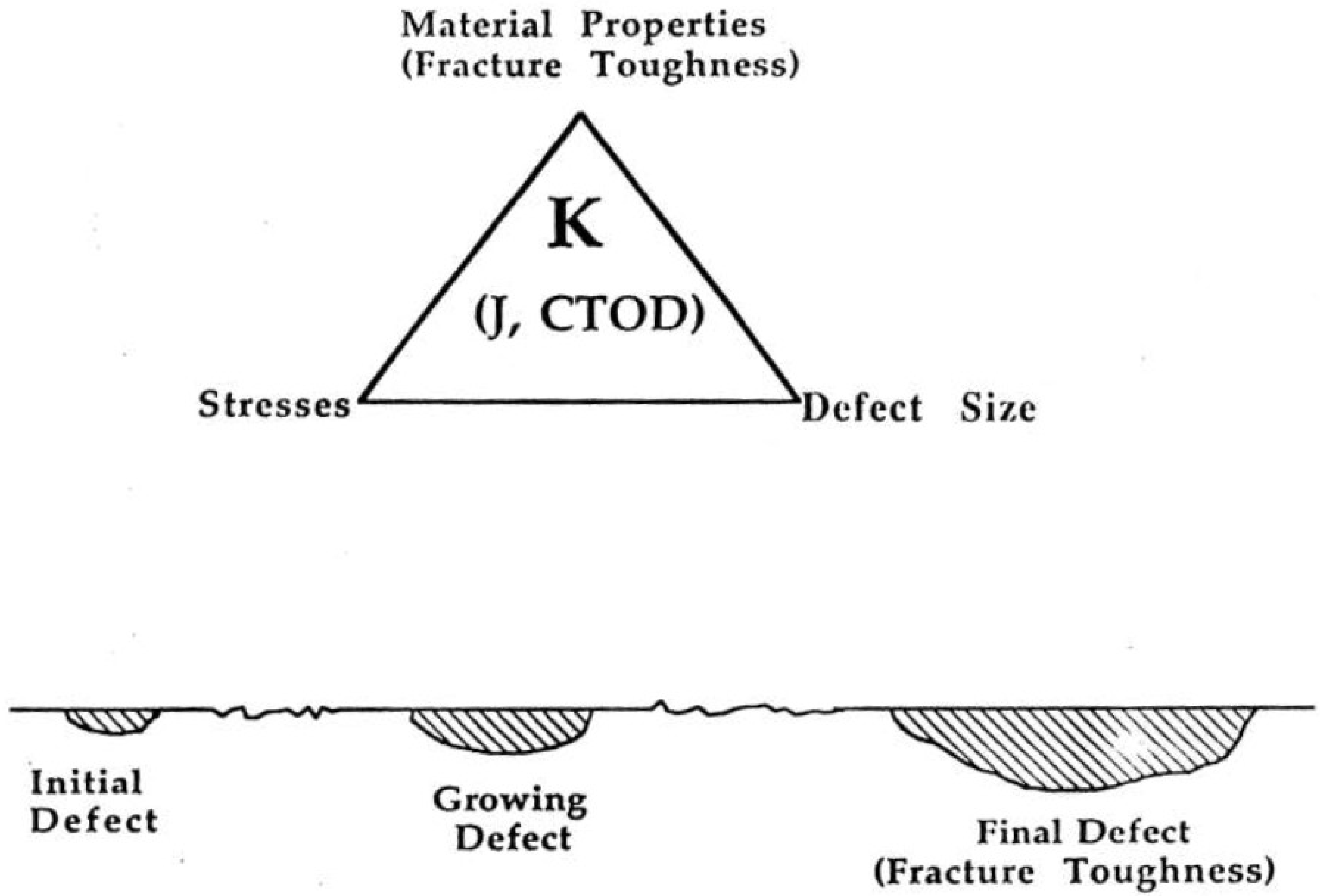
η is a function of the aspect ratio, a/W

$$\eta = 3.785 - 3.101 \cdot \frac{a}{W} + 2.018 \cdot \left(\frac{a}{W}\right)^2. \quad (5$$

At the Atlanta Task Group meeting, May 1998, it was agreed that the above expressions for m



(a) DESIGN CURVE FOR CTOD, STRAIN, AND CRACK SIZE RELATIONSHIP AS PROPOSED BY BURDEKIN AND DAWES



Fracture Toughness for Structural Integrity