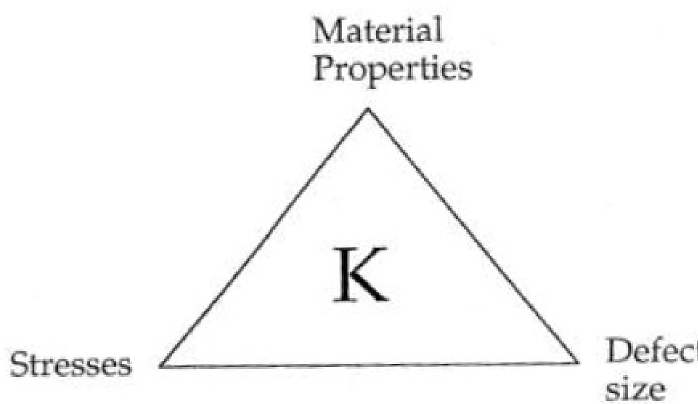
Section 7

Linear-elastic FM applications

Applications

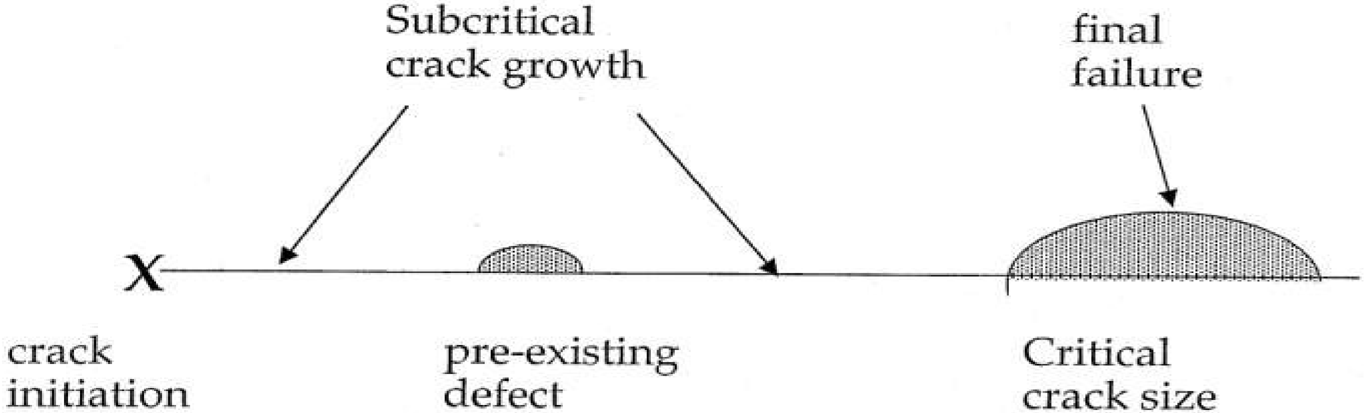
1. Reasons for applications
   - Design criteria
   - Material selection
   - Failure Analysis

2. Use the FM triangle as a guide

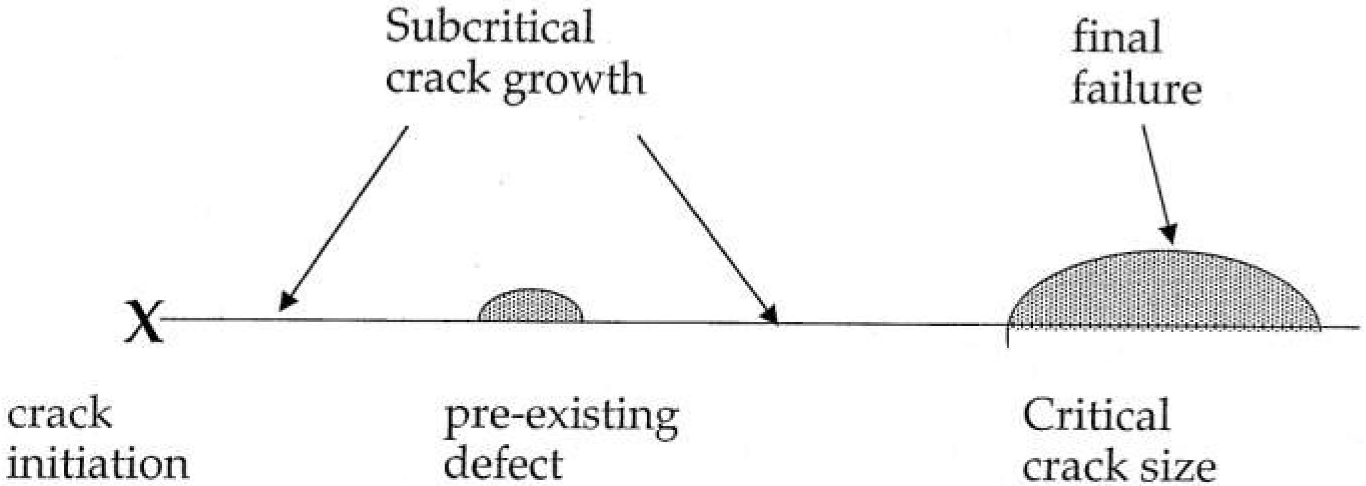


Given two corners, predict the third

Schematic of Life Prediction



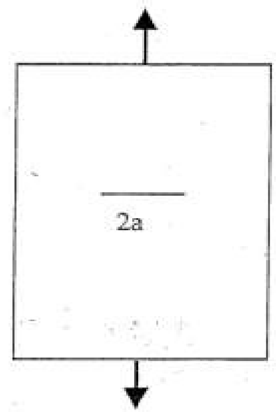
Life of a structure containing a crack-like defect



Life of a structure containing a crack-like defect

K_{Ic} Application

1. Given a wide plate with a central crack



K_{Ic} from Workshop problem, $K_{Ic} = 81$ ksi$\sqrt{\text{in}}$

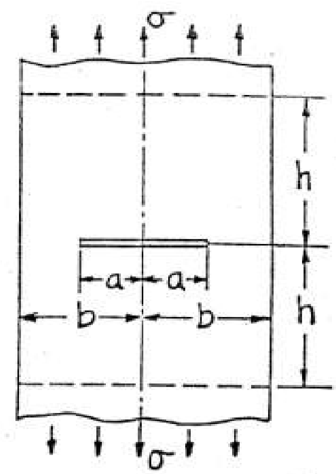
1. Given a defect size, 2a = 3 in, what is σ_{cr}?
   (assume an infinite plate)

$$2a = 3 \text{ in},\ a = 1.5 \text{ in}$$
$$K = \sigma\sqrt{(\pi a)}$$
$$81 = \sigma_{cr}\sqrt{(1.5\pi)}$$
$$\sigma_{cr} = 37.3 \text{ ksi}$$

THE CENTER CRACKED TEST SPECIMEN

A. Stress Intensity Factor

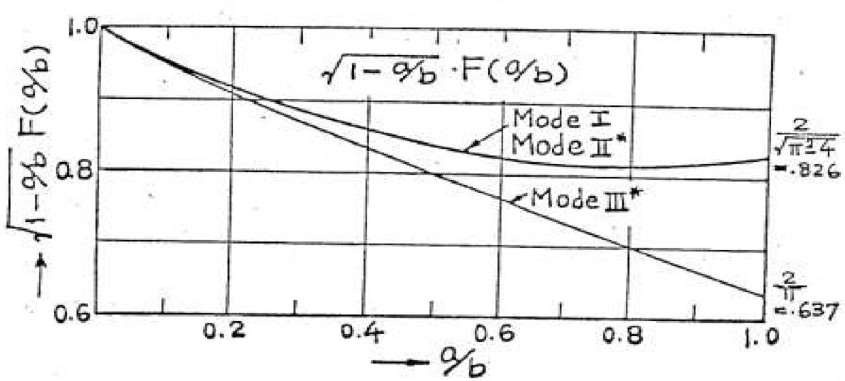
$$K_I = \sigma\sqrt{\pi a}\, F(a/b)$$



Numerical Values at F(a/b)

(Isida 1962, 1965 a, b, 1973)

Isida's 36 term power series of $(a/b)^2$ (Laurent series expansion of complex stress potential, 1973) gives practically exact values of F(a/b) up to a/b = 0.9. Numerical values of F(a/b) are shown in the table.



a/b	F(a/b)
0.0	1.0000
0.1	1.0060
0.2	1.0246
0.3	1.0577
0.4	1.1094
0.5	1.1867
0.6	1.3033
0.7	1.4882
0.8	1.8160
0.9	2.5776
1.0	$\frac{2}{\sqrt{\pi^2-4}}\Big/\sqrt{1-a/b}$ **

2. Given $\sigma = 25$ ksi, what is a_{cr}?

$$81 = 25\sqrt{(\pi a_{cr})}$$

$a_{cr} = 3.34$ in; defect size is $2a_{cr} = 6.68$ in.

3. Now assume a finite plate

$2b = 10$ in., $B = 1$ in., $\sigma = 25$ ksi.

Load, $P = (25\text{ksi})(10\text{x}1)\ \text{in}^2 = 250$ kips

What is a_{cr}?

$K = \sigma\sqrt{(\pi a)}F(a/b)$;

a at two places so cannot solve directly

If $a = 3.34$, $a/b = 3.34/5 = 0.67$, $F = 1.42$

$K = 25\sqrt{(3.34\pi)}(1.42) = 115$ ksi$\sqrt{\text{in}}$; too large

i) Try $2a = 2.5$, $a/b = 0.5$, $F = 1.187$, $K = 83.2$; close

ii) Try $a= 2.4$, $a/b = .48$, $F = 1.179$, $K = 80.9$, good

Then, $a_{cr} = 2.4$ in., $2a_{cr} = 4.8$ in.

Predicting life from da/dN vs ΔK

1. $da/dN = C(\Delta K)^n$

2. $K = \sigma\sqrt{\pi a}\, F$

3. $da/dN = C(\Delta\sigma\sqrt{\pi a}\, F)^n$

4. $dN = da/C(\Delta\sigma\sqrt{\pi a}\, F)^n$

5.

$$N = \int_{a_o}^{a_f} \frac{da}{C\,(\Delta\sigma\sqrt{\pi a}\, F)^n}$$

6. This is easier if F is constant or a simple function

da/dN vs ΔK Application

Same wide plate with a central crack model

Load: 0 to 25 ksi; $a_o = 0.1$ in, inspection;

K_{Ic} from before makes $a_f = 3.34$ in.

Assume CGR law: $da/dN = 1x10^{-9}\Delta K^3$

How many cycles to fail, N_f?

Solution:

$$\Delta K = \Delta\sigma\sqrt{\pi a} = 25\sqrt{\pi a}$$

$$da/dN = 1x10^{-9}(25\sqrt{\pi a})^3 = 8.70x10^{-5}a^{3/2}$$

Rearrange:

$$dN = \frac{da}{8.70x10^{-5}a^{3/2}}$$

$$N_f = \int_{a_o}^{a_f} \frac{da}{(*)} = \int_{0.1}^{3.34} \frac{da}{8.70x10^{-5}a^{3/2}} = \frac{1}{8.70x10^{-5}}\left[\frac{a^{-1/2}}{(-1/2)}\right]_{0.1}^{3.34}$$

$$N_f = \frac{2}{8.70x10^{-5}}\left[\frac{1}{\sqrt{0.1}} - \frac{1}{\sqrt{3.34}}\right] = 6.01x10^4 \approx 60,000$$

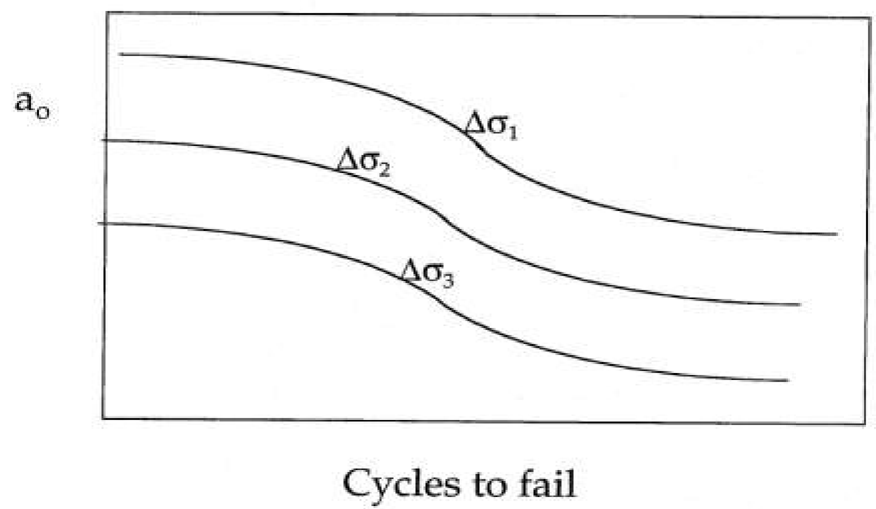
Fatigue crack growth lives

1. Start with test specimen

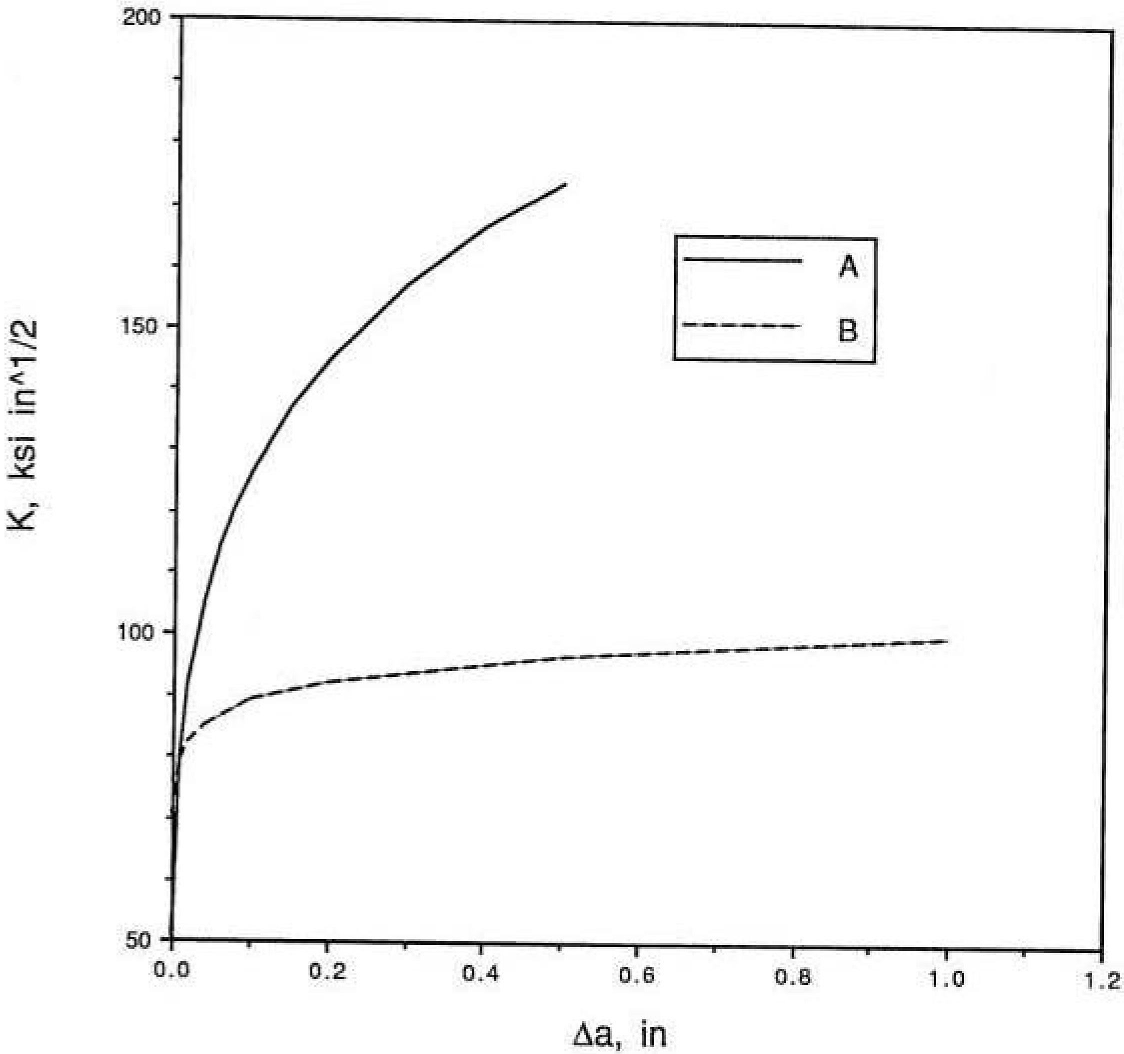
2. Generate da/dN vs ΔK for a range of K

3. For given stress range, a_o, a_f (K_{Ic}), evaluate lives

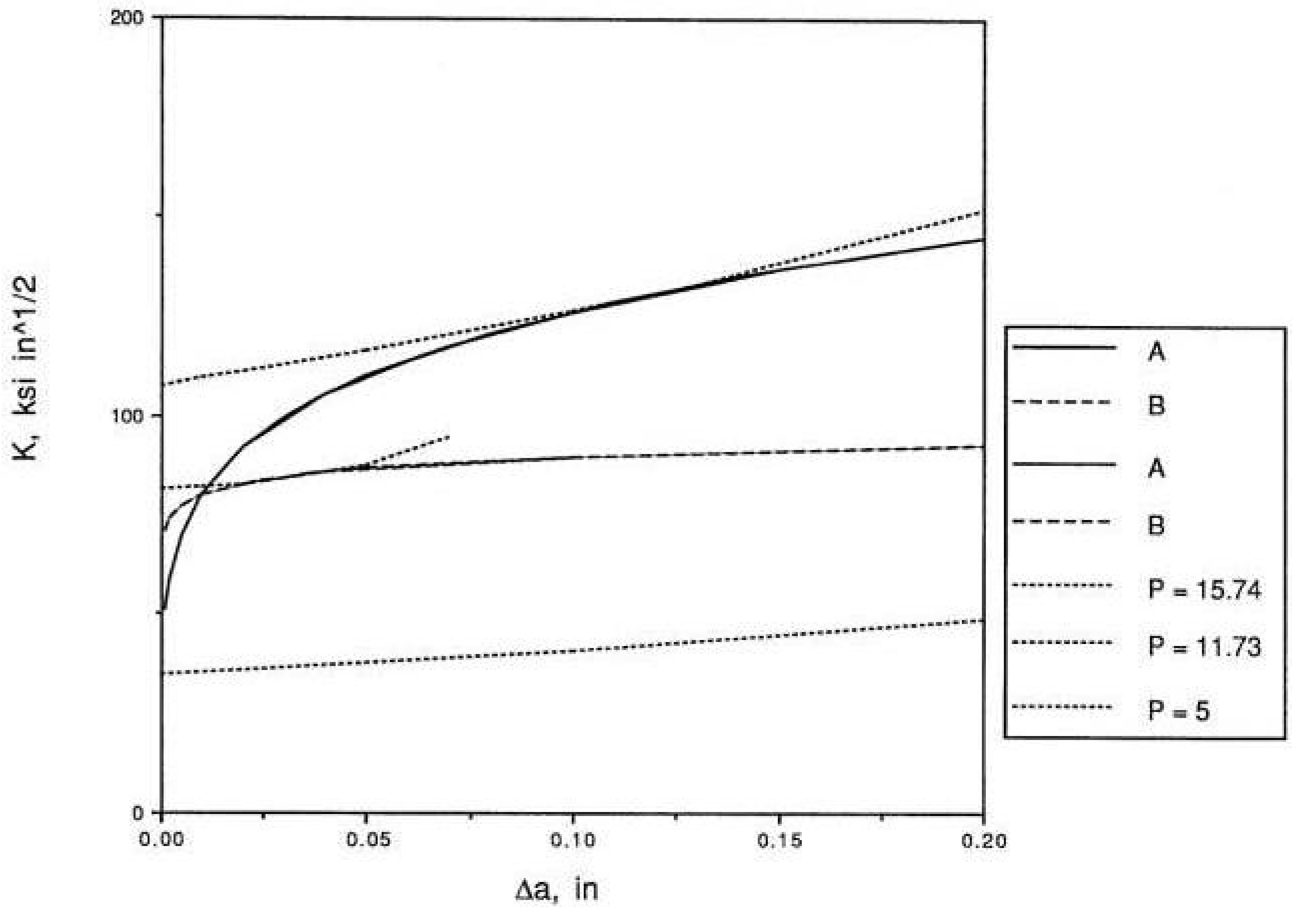
4. If design parameters are not precisely known, develop a parametric set of curves for a range of input

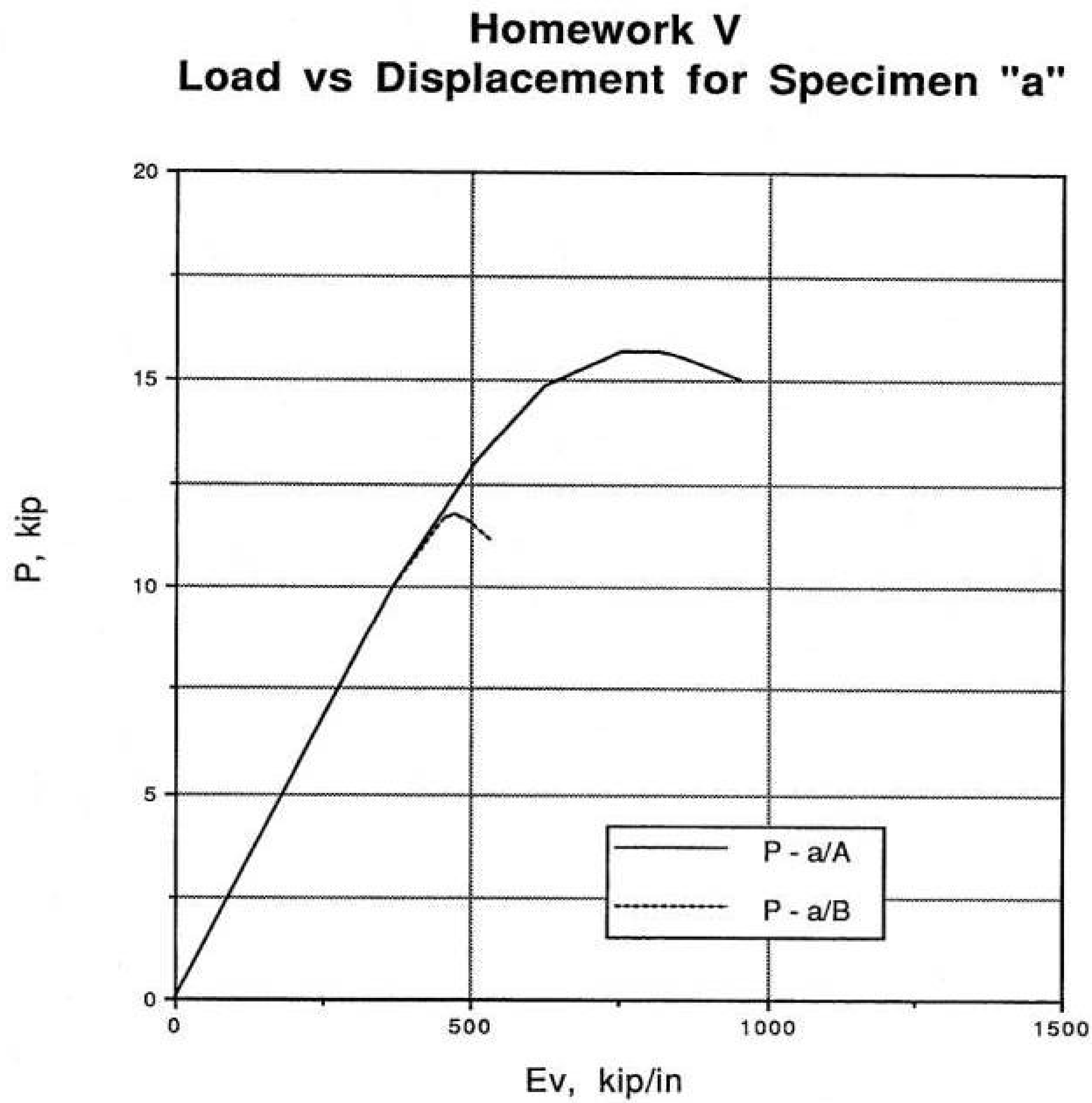


Homework 5 K-R Curve



Homework 5 K-R Curve Instability




Homework V
Load vs Displacement for Specimen "a"
P, kip
Ev, kip/in
0
5
10
15
20
0
500
1000
1500
P - a/A
P - a/B

Special Application Methods in FM

1. "Leak before break"

   a) Assume that a surface flaw grows through the wall with a known pattern, this is the leak condition

   b) Assume surface crack length at leaking, 2c, is 2a, in a CCT specimen and see if K exceeds K_{Ic}.

Problems: It is difficult to predict the correct pattern for growth, especially for bending loads

2. Proof Testing

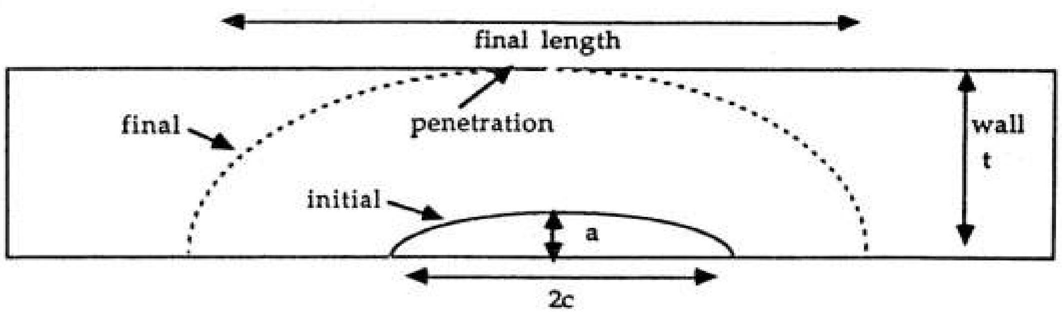
   a) Load to a stress higher than operating

   b) If there is no failure, assume that failure is impending and a_o comes from K_{Ic}.

   c) This gives the a_o for FM analysis at the operating stress, e. g. due to SCC, da/dN, etc.

Leak Before Break (LBB)

1. Part-through crack grows in pressure boundary wall
   - Will it grow through and leak (LBB)? or
   - Will it burst ?

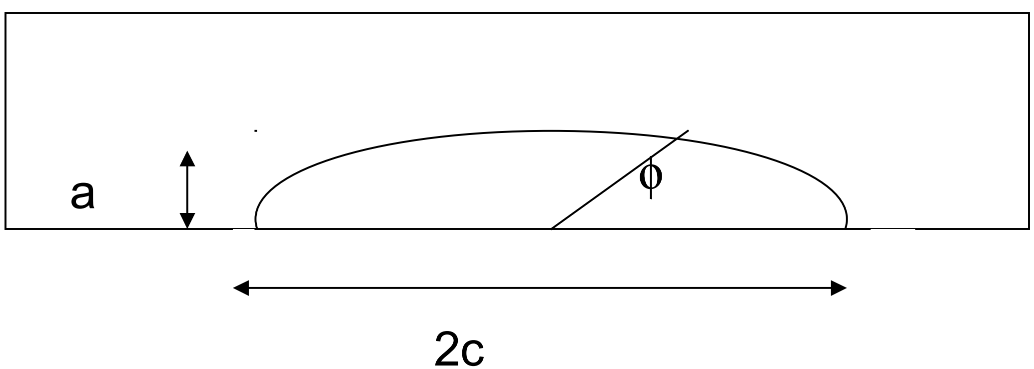


2. Critical considerations:
   - Final length of crack as it penetrates
   - Toughness
   - Shape crack assumes as it grows

Part-through crack K calculations - N

- K expression under tensile stress

$$K = 1.122\sigma\sqrt{\frac{\pi a}{Q}}\left[\sin^2\phi + \left(\frac{a}{c}\right)^2\cos^2\phi\right]^{1/4} \qquad Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$



K calculation (cont.)

- Let a = 1 in, 2c = 6 in, σ = 30 ksi

- a/2c = 0.167, $Q = 1 + 1.464(0.333)^{1.65} = 1.249$

$$K = 1.122(30)\sqrt{\frac{\pi(1)}{1.239}}\left[\sin^2\phi + (.333)^2\cos^2\phi\right]^{1/4}$$

$$= 53.6\left[\sin^2\phi + .111\cos^2\phi\right]^{1/4}$$

- K is a function of position given by ϕ

K calculation (cont) - N

- Let $\phi = 90°$, $K = 56.3(1)$
  (deepest part of crack)

- Let $\phi = 0°$, $K = 56.3(.111)^{1/4} = 32.5$
  (Surface edge of crack)

As a consequence the deepest part of the crack grows faster under fatigue; $da/dN = C\Delta K^n$

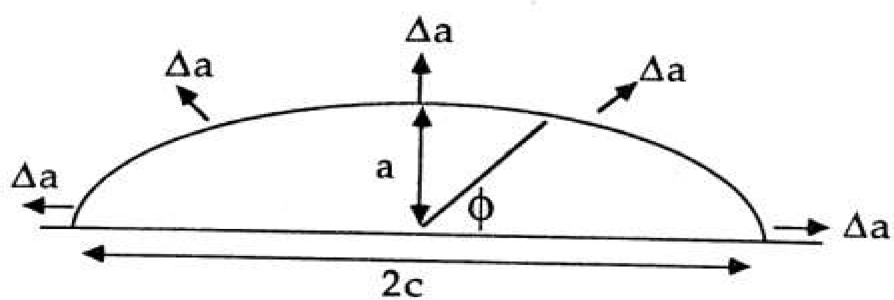
Crack tries to become semi-circular

Part Through Crack Growth

1. The K around a part through crack is a function of angle, ϕ

2. The crack grows at a different rate as a function of position

3. The crack shape change can be calculated by considering the growth at individual points, local Δa, for individual K values over a ΔN increment



$K = K(\phi)$

4. Since only elliptical cracks shapes have K calibrations, the Δa and Δc could be measured and a new elliptical crack shape calculated

Stress-Intensity Factors for Internal and External Surface Cracks in Cylindrical Vessels

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NASA Langley Research Center, Hampton, Virginia 23665

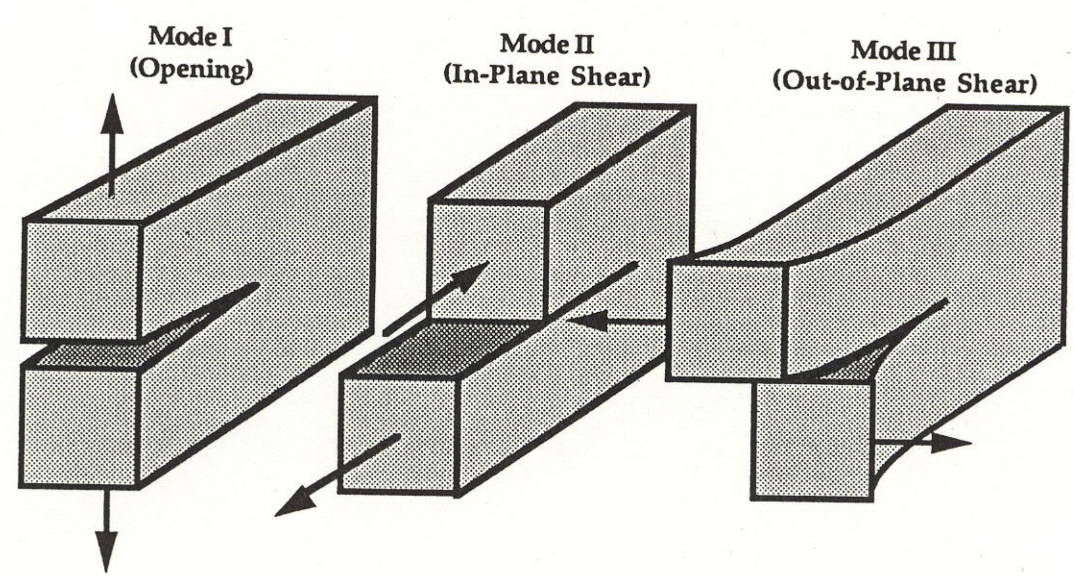
The purpose of this paper is to present stress-intensity factor influence coefficients for a wide range of semi-elliptical surface cracks on the inside or outside of a cylinder. The crack surfaces were subjected to four stress distributions: uniform, linear, quadratic, and cubic. These four solutions can be superimposed to obtain stress-intensity factor solutions for other stress distributions, such as those caused by internal pressure and by thermal shock. The results for internal pressure are given herein. The ratio of crack depth to crack length from 0.2 to 1; the ratio of crack depth to wall thickness ranged from 0.2 to 0.8; and the ratio of wall thickness to vessel radius was 0.1 or 0.25. The stress-intensity factors were calculated by a three-dimensional finite-element method. The finite-element models employ singularity elements along the crack front and linear-strain elements elsewhere. The models had about 6500 degrees of freedom. The stress-intensity factors were evaluated from a nodal-force method. The present results were also compared to other analyses of surface cracks in cylinders. The results from a boundary-integral equation method agreed well (±2 percent), and those from other finite-element methods agreed fairly well (±10 percent) with the present results.

Mixed Mode FM

- Sometimes the crack is not aligned perpendicular to the loading direction
- This can cause a component of Mode II loading
- Mixed mode or Mode II loading is studied about every 15 years

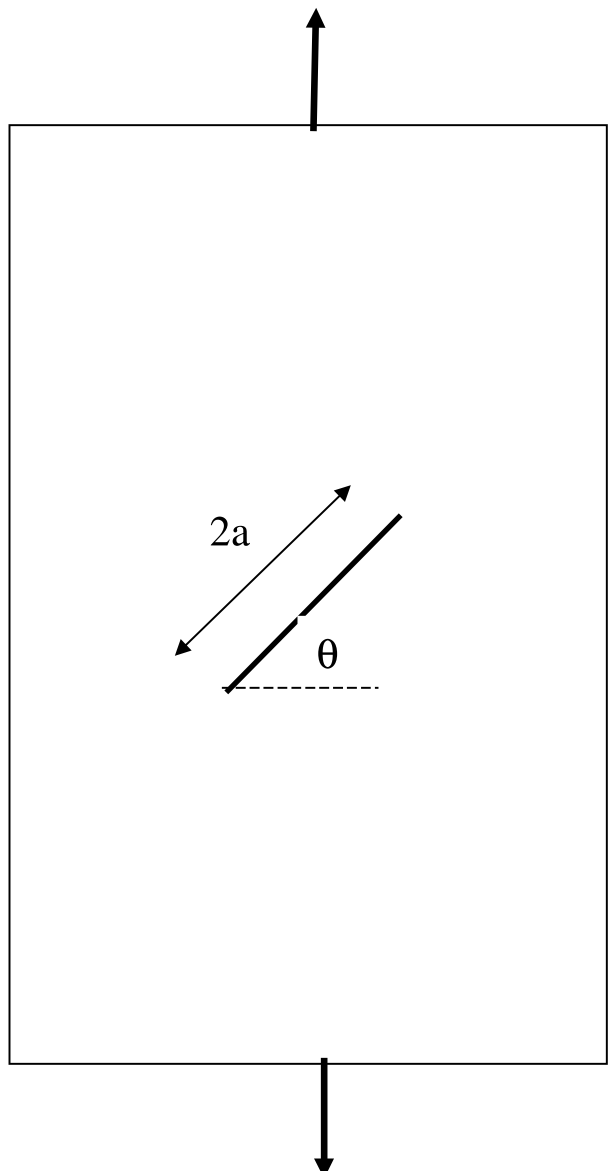
Modes of loading - N

A crack can be loaded in one of three ways:



The stress fields ahead of a crack tip in a linear elastic material are proportional to $1/\sqrt{r}$. The proportionality constant is called the *stress intensity factor.*

Mixed mode load - N



K solutions - N

$$K_I = \sigma\sqrt{\pi a}\cos^2\theta$$

$$K_{II} = \sigma\sqrt{\pi a}\cos\theta\sin\theta$$

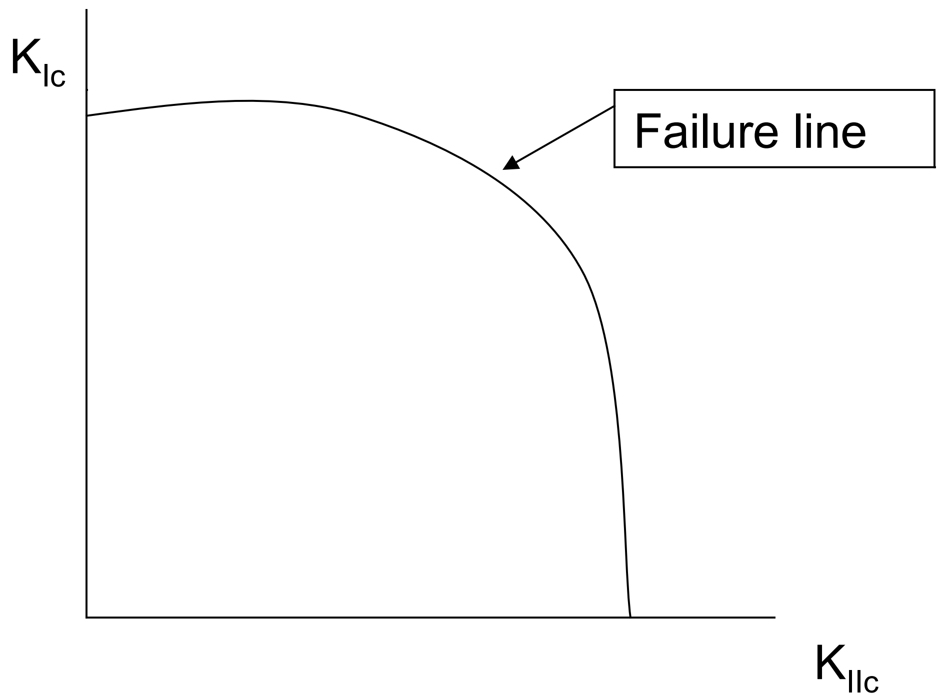
Combining Mixed Mode is by Griffith G - N

$$G_c = \frac{K_{Ic}^2 + K_{IIc}^2}{E}$$

Determining combined toughness - N

- K_{Ic} is usually the only toughness measured
- K_{IIc} when measured was usually greater than K_{Ic}
- A circular failure locus could be conservative
- When in doubt measure K_{IIc} and us an elliptical failure locus

Failure locus - N



ASME Boiler and Pressure Vessel Code -

Use of Fracture Mechanics Concepts

1. Section III - Rules for Construction of Nuclear Power Plant Components

Appendix G - Protection Against Nonductile Fracture

- Use of the K_{IR} curve

2. Section XI - Rules for Inservice Inspection of Nuclear Power Plant Components

Appendix A - Analysis of Flaws (nonmandatory)

- K_{Ic} - K_{IR} Curves
- da/dN vs ΔK
- Crack Arrest

ARTICLE G-2000
VESSELS

G-2100 GENERAL REQUIREMENTS

G-2110 REFERENCE CRITICAL STRESS INTENSITY FACTOR

(a) Figure G-2210-1 is a curve showing the relationship that can be conservatively expected between the critical, or reference, stress intensity factor K_{IR}, ksi$\sqrt{\text{in.}}$, and a temperature which is related to the reference nil ductility temperature RT_{NDT} determined in NB-2331. This curve is based on the lower bound of static, dynamic, and crack arrest critical K_I values measured as a function of temperature on specimens of SA-533 Grade B Class 1, and SA-508-1, SA-508-2, and SA-508-3 steel. No available data points for static, dynamic, or arrest tests fall below the curve. An analytical approximation to the curve is:

$$K_{IR} = 26.78 + 1.223\exp[0.0145(T - RT_{NDT} + 160)]$$

Unless higher K_{IR} values can be justified for the particular material and circumstances being considered, Fig. G-2210-1 may be used for ferritic steels which meet the requirements of NB-2331 and which have a specified minimum yield strength at room temperature of 50.0 ksi or less.

(b) For materials which have specified minimum yield strengths at room temperature greater than 50.0 ksi but not exceeding 90.0 ksi, Fig. G-2210-1 may be used provided fracture mechanics data (similar to the K_{ID} data referenced in WRCB 175) are obtained on at least three heats of the material on a sufficient number of specimens to cover the temperature range of interest, including the weld metal and heat-affected zone, and provided that the data are equal to or above the curve of Fig. G-2210-1. These data shall be included in the Design Specification. Where these materials of higher yield strengths (specified minimum yield strength greater than 50.0 ksi but not exceeding 90.0 ksi) are to be used in conditions where radiation may affect the material properties, the effect of radiation on the K_{IR} curve shall be determined for the material prior to its use in manufacture. This information shall be included in the Design Specification.

G-2120 MAXIMUM POSTULATED DEFECT

The postulated defect used in this recommended procedure is a sharp, surface defect normal to the direction of maximum stress. For section thicknesses of 4 in. to 12 in., it has a depth of one-fourth of the section thickness and a length of $1^1/_2$ times the section thickness. For sections greater than 12 in. thick, the postulated defect for the 12 in. section is used. For sections less than 4 in. thick, the 1 in. deep defect is conservatively postulated. These postulated effects of thickness were used in developing the curves of Fig. G-2214-1. Smaller defect sizes[1] may be used on an individual case basis if a smaller size of maximum postulated defect can be ensured. Due to the safety factors recommended here, the prevention of nonductile fracture is ensured for some of the most important situations even if the defects were to be about twice as large in linear dimensions as this postulated maximum defect.

G-2200 LEVEL A AND LEVEL B SERVICE LIMITS

G-2210 SHELLS AND HEADS REMOTE FROM DISCONTINUITIES

G-2211 Recommendations

The assumptions of this Subarticle are recommended for shell and head regions during Level A and B Service Limits.

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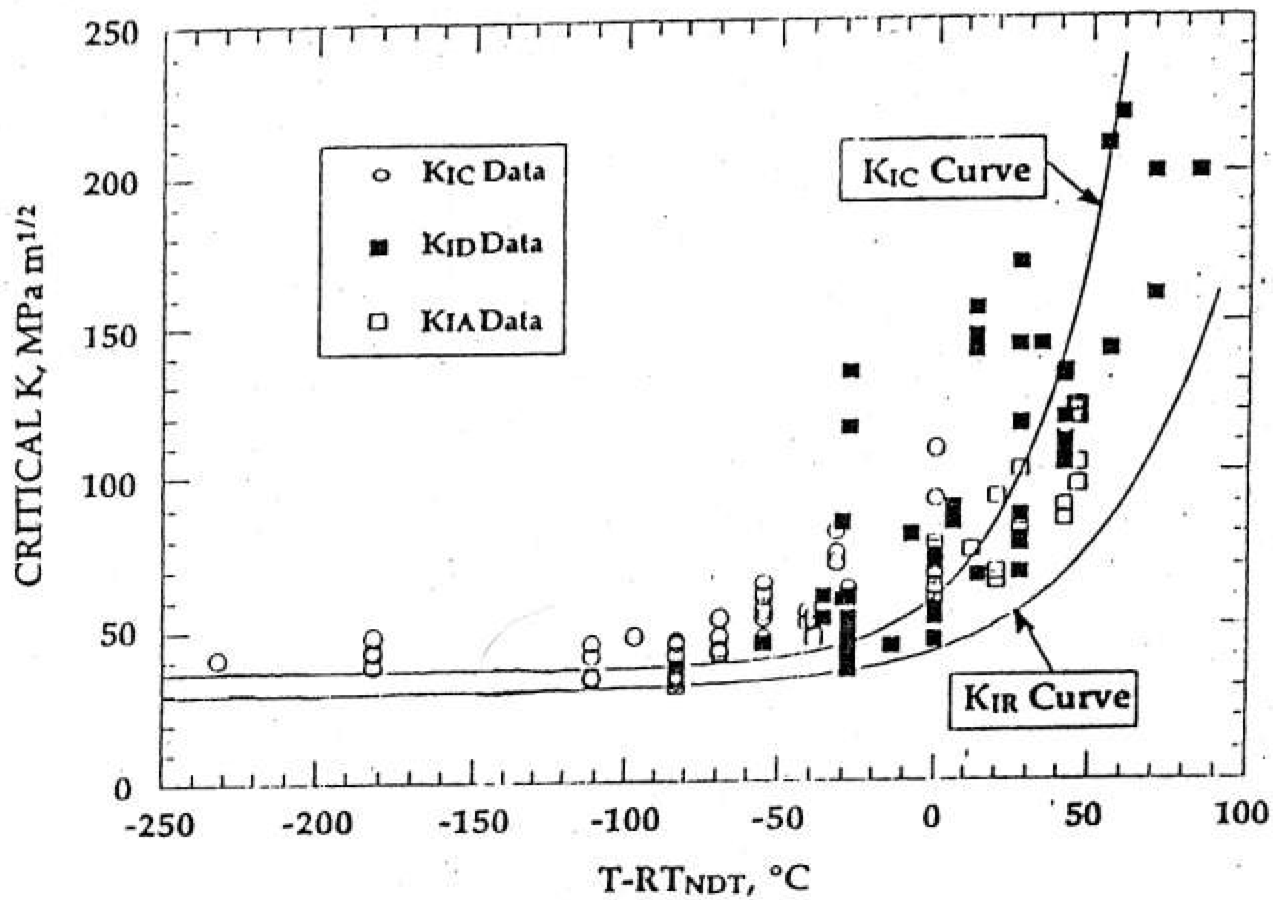
[1] WRCB 175 (Welding Research Council Bulletin 175) "PVRC Recommendations on Toughness Requirements for Ferritic Materials" provides procedures in Paragraph 5(c)(2) for considering maximum postulated defects smaller than those described.

ASME Section Boiler and Pressure Vessel Codes

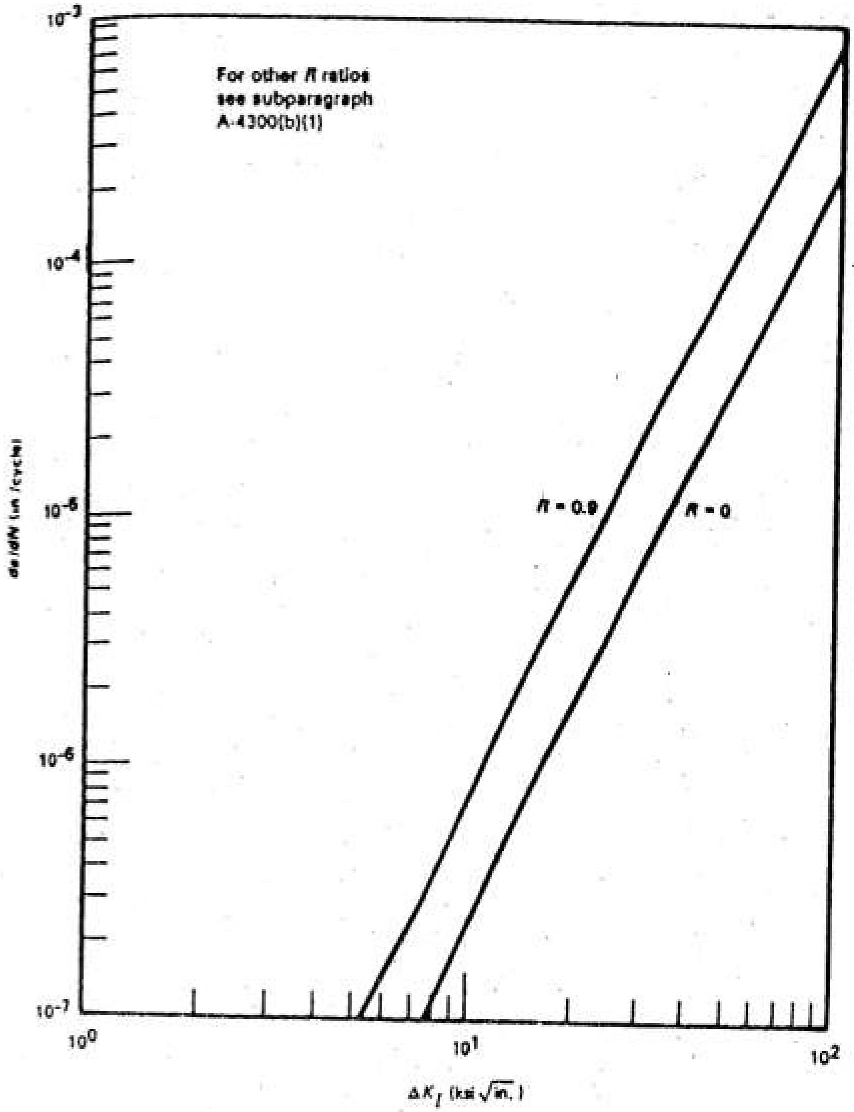
Reference Curves

$$K_{IC} = 36.5 + 3.084 \exp[0.036(T - RT_{NDT} + 56)]$$

$$K_{IR} = 29.5 + 1.344 \exp[0.026(T - RT_{NDT} + 89)]$$



KIC and KIR Curves From ASME Section XI



4300-1 REFERENCE FATIGUE CRACK GROWTH CURVES FOR CARBON AND LOW ALLOY FERRITIC STEELS EXPOSED TO AIR ENVIRONMENTS (SUBSURFACE FLAWS)

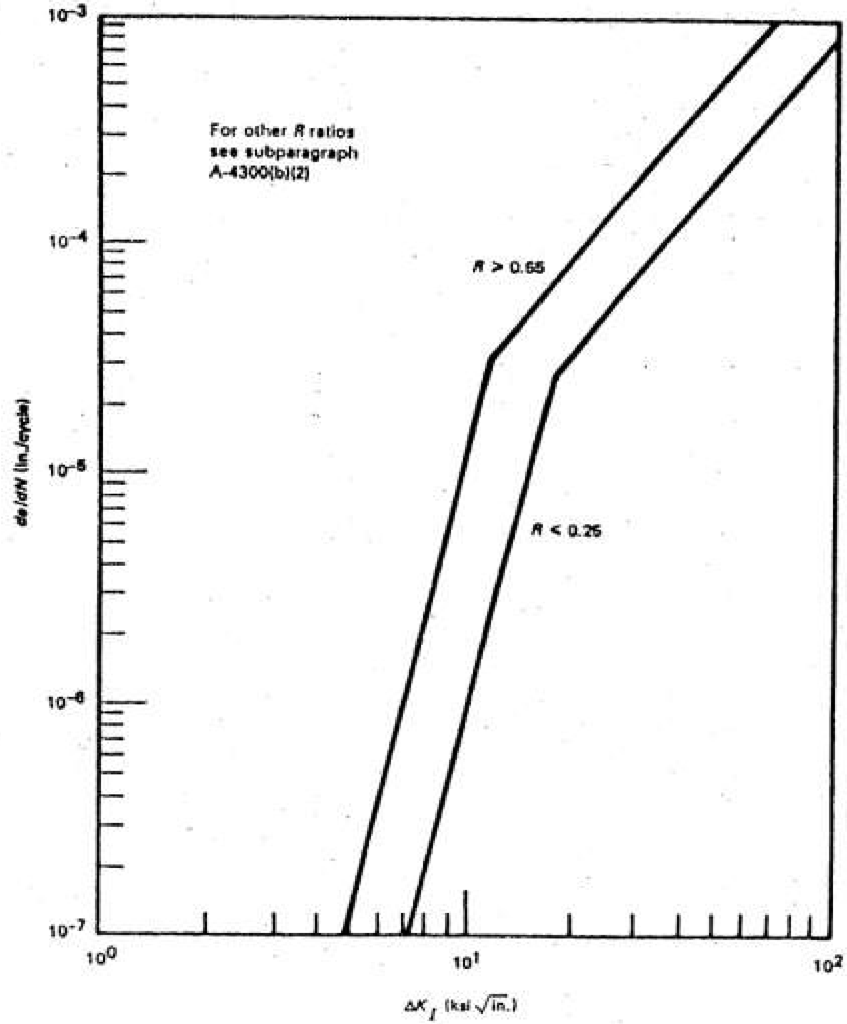


FIG. A-4300-2 REFERENCE FATIGUE CRACK GROWTH CURVES FOR CARBON AND LOW ALLOY FERRITIC STEELS EXPOSED TO WATER ENVIRONMENTS