

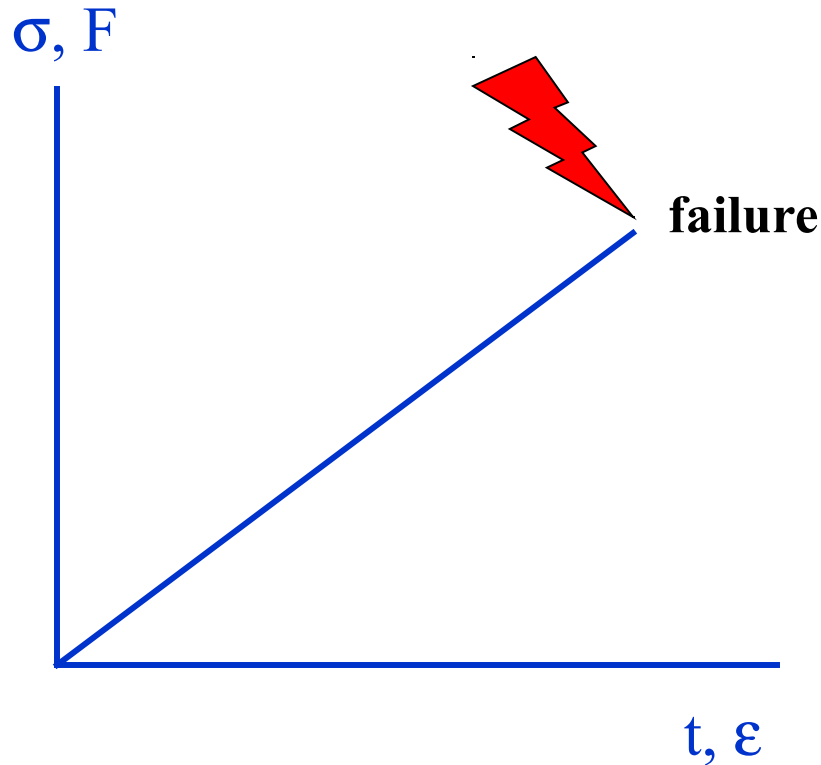
# Lecture 5

## Fatigue Concepts

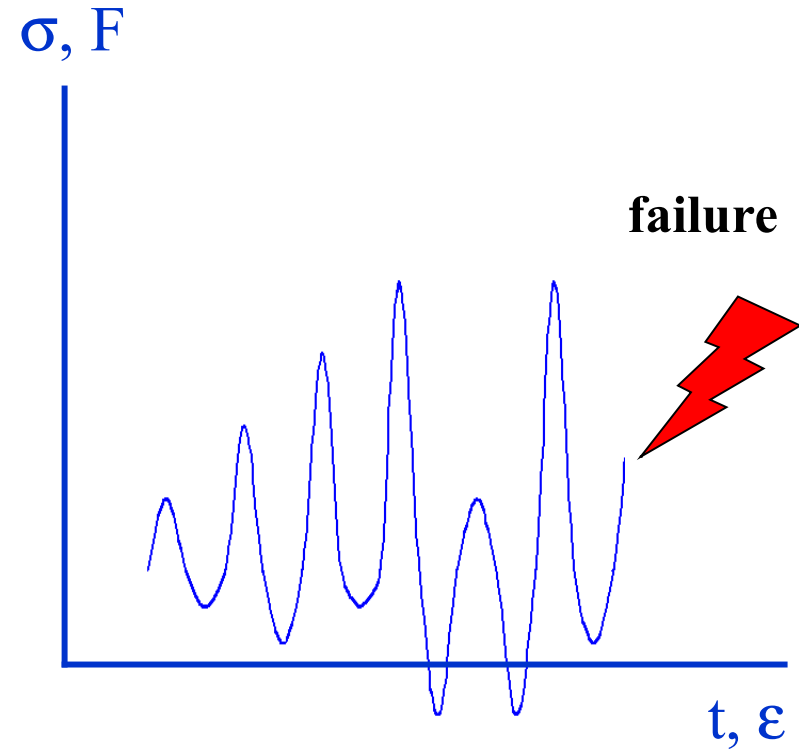
# Fatigue

1. Cyclic loading leads to failure – crack-like defects initiate and grow to failure
1. Economic Impact in the USA > \$100B/yr
3. Failure examples: airplanes, turbines  
bridges

# Fatigue versus monotonic load failure

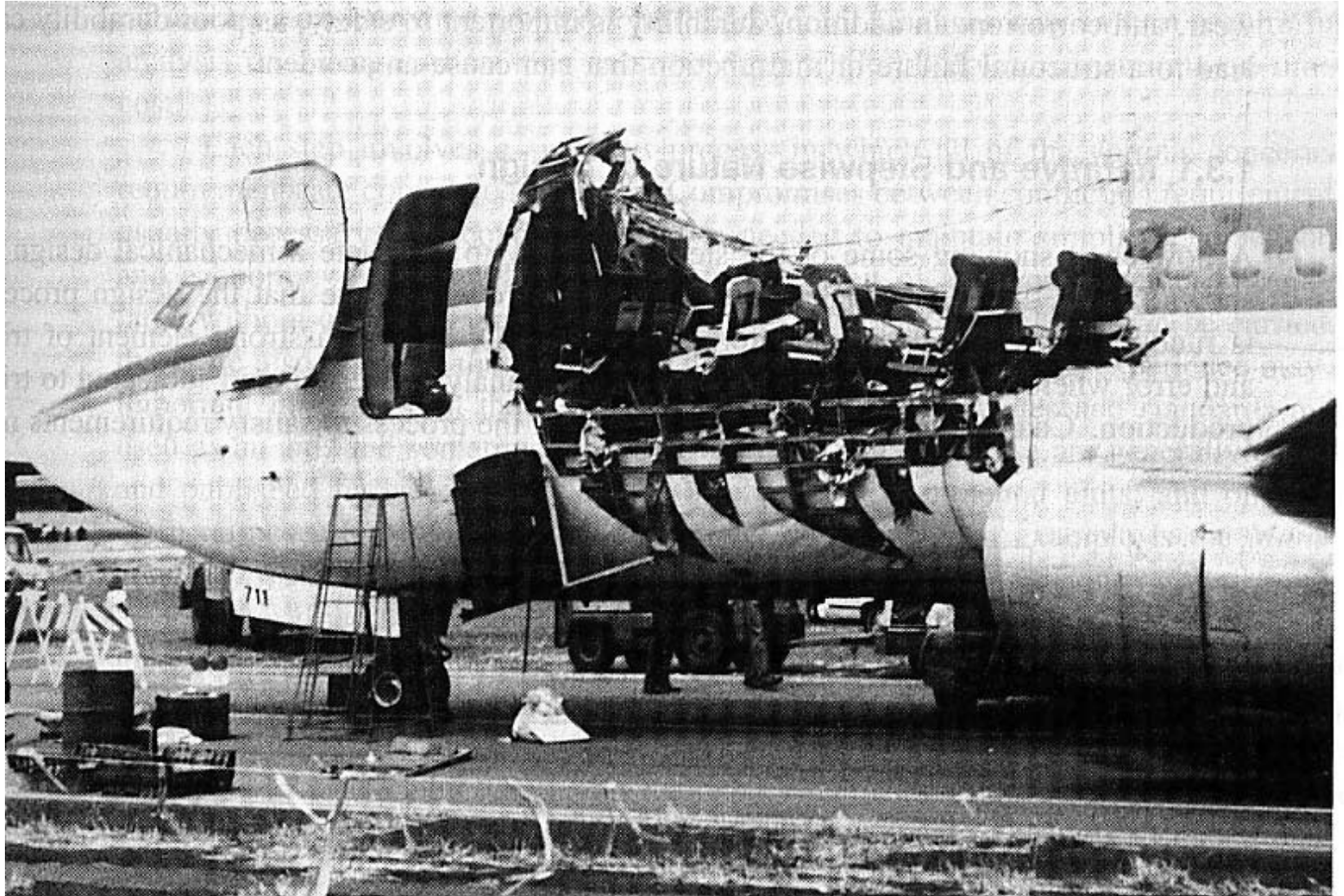


Monotonic loading



Cyclic loading

# Aloha aircraft



# Approaches

- High cycle – stress versus life plot (S-N)
- Low cycle – Strain versus life plot ( $\epsilon - N_f$ )
- Crack growth rate –  $da/dN$  versus  $\Delta K$   
(this is a fracture mechanics approach)

## High Cycle Fatigue Stress – Life, S–N, approach

Specimen is subjected to cyclic loading

- Stress amplitude is imposed,  $\sigma_a$
- Cycles to fail is measured,  $N_f$
- Result is a point on the S-N plot

# S-N Curve

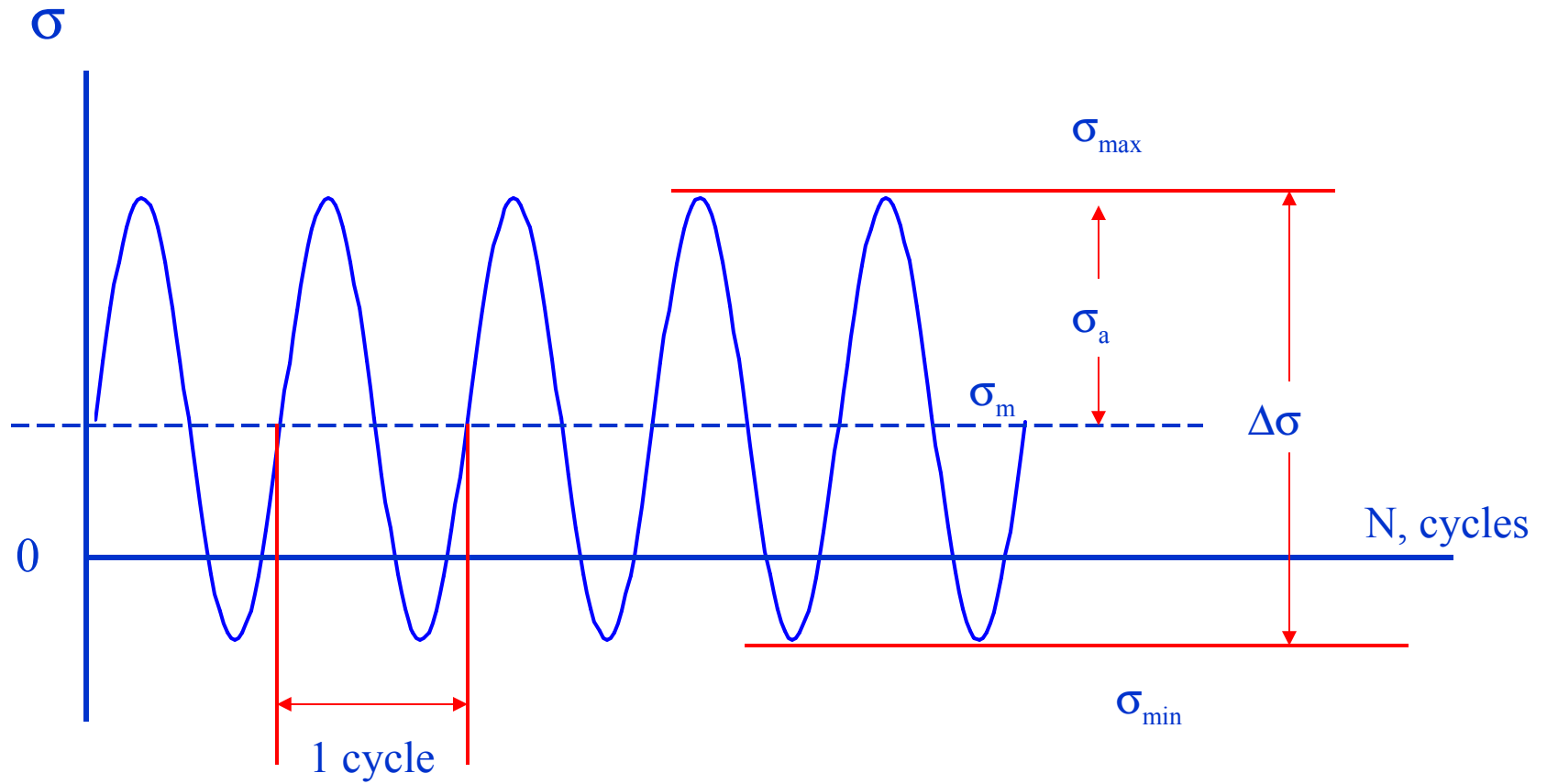
- 8 to 10 specimens are run to create an S-N curve
- Points on the S-N curve can be fitted
  - Semi-log;  $\sigma_a = C + D \log N_f$
  - log-log;  $\sigma_a = A (N_f)^B$

# S-N Curve

- 8 to 10 specimens are run to create an S-N curve
- Points on the S-N curve can be fitted
  - Semi-log;  $\sigma_a = C + D \log N_f$
  - log-log;  $\sigma_a = A (N_f)^B$



# Stress – life Nomenclature

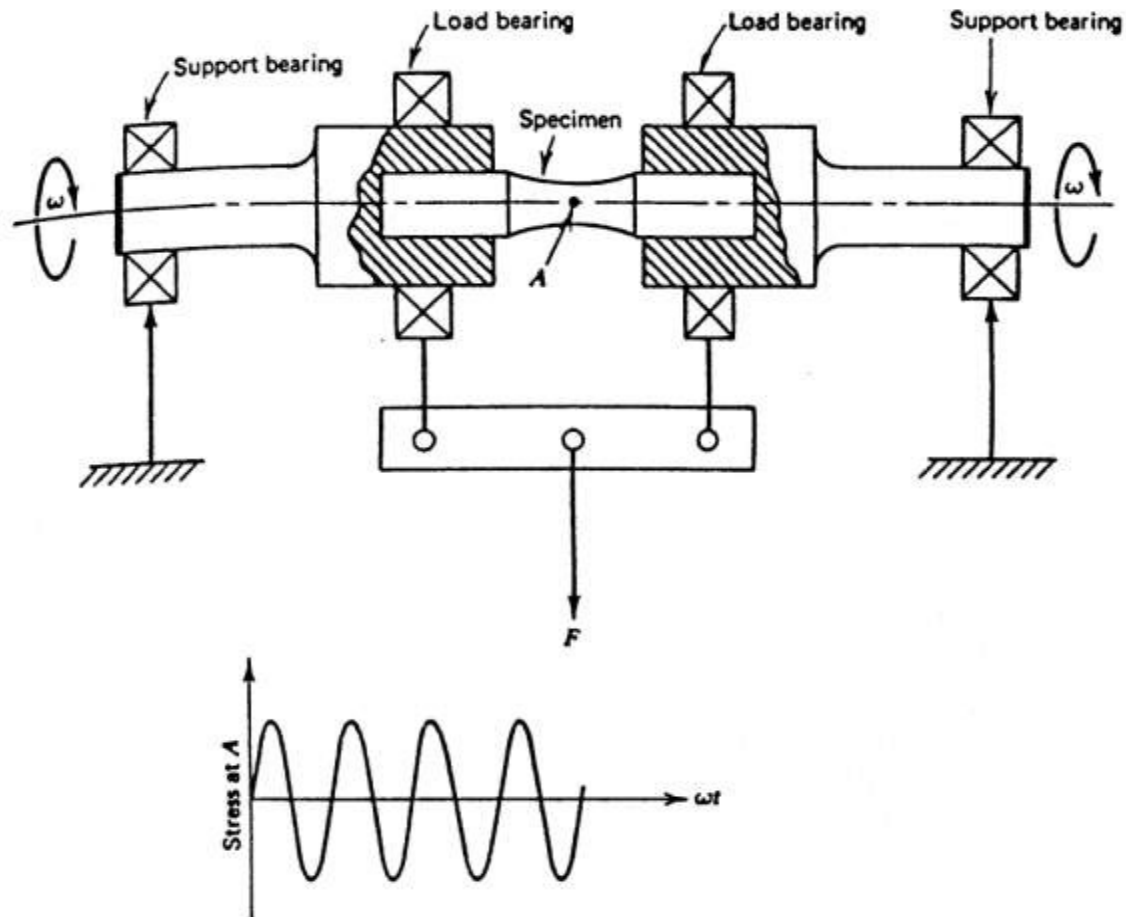


# Nomenclature

- $\sigma_{\max}$  = maximum stress
- $\sigma_{\min}$  = minimum stress
- $\Delta\sigma$  = stress range =  $\sigma_{\max} - \sigma_{\min}$
- $\sigma_a$  = stress amplitude =  $\Delta\sigma/2$
- $\sigma_m$  = mean stress =  $(\sigma_{\max} + \sigma_{\min})/2$
- $R$  = stress ratio =  $\sigma_{\min}/\sigma_{\max}$

# Test methods

1. Rotating bend –  $\sigma_m$  is always zero
2. Reversed bend –  $\sigma_m$  can be adjusted
3. Axial, same as 2 but more scientific approach



**FIGURE 7.7.** Rotating-bending fatigue testing machine of the constant bending moment type.

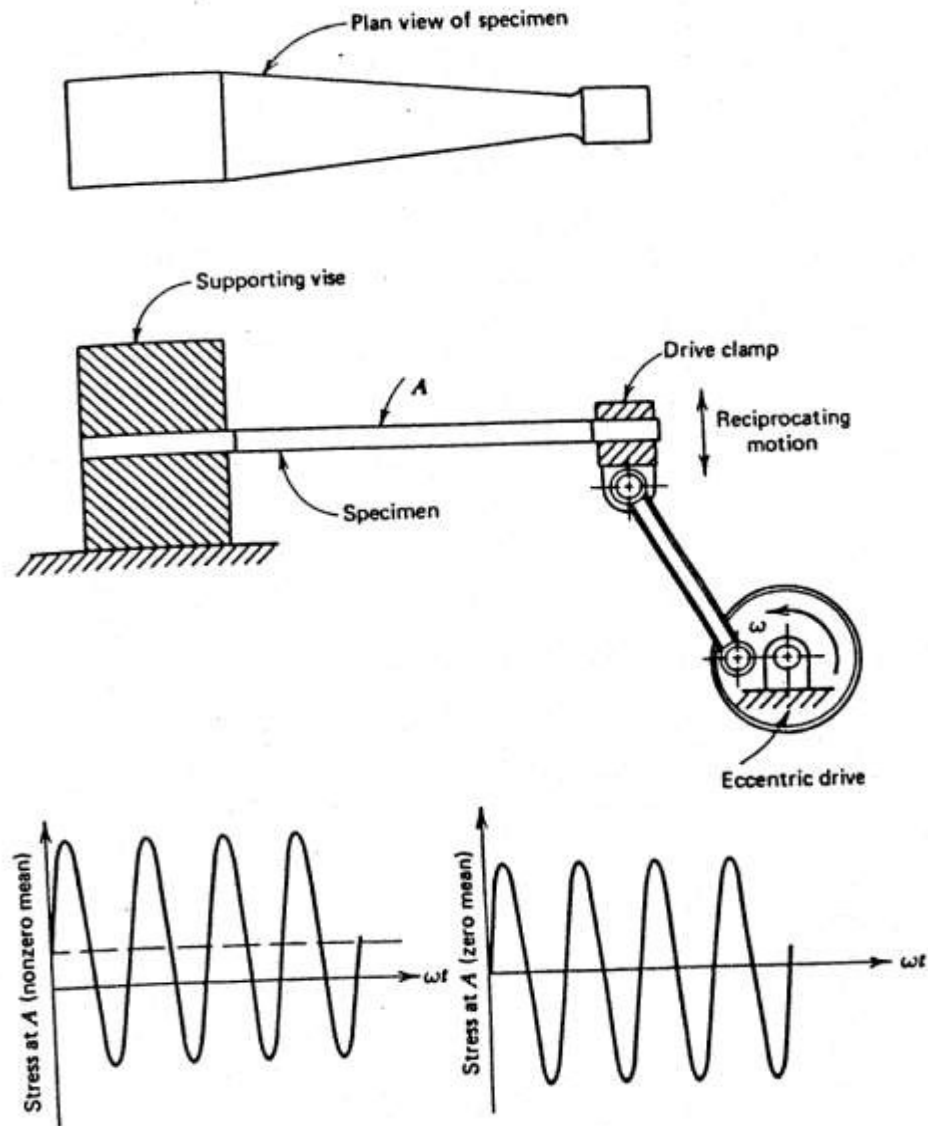


FIGURE 79. Reciprocating-bending fatigue testing machine.

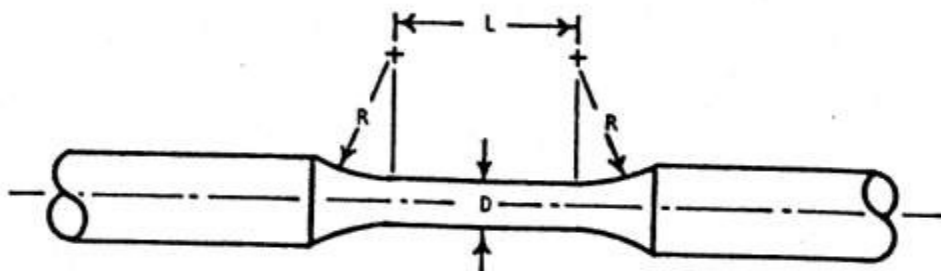


FIG. 1 Specimens with Tangentially Blending Fillets Between the Test Section and the Ends

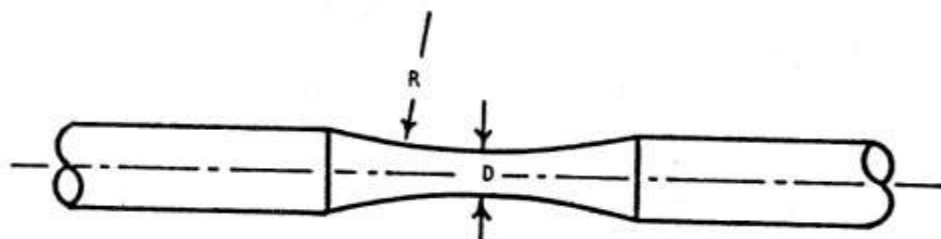


FIG. 2 Specimens with a Continuous Radius Between Ends

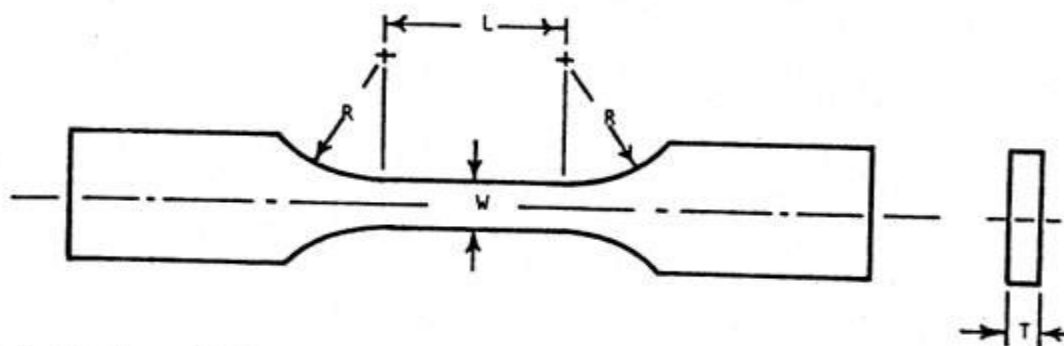
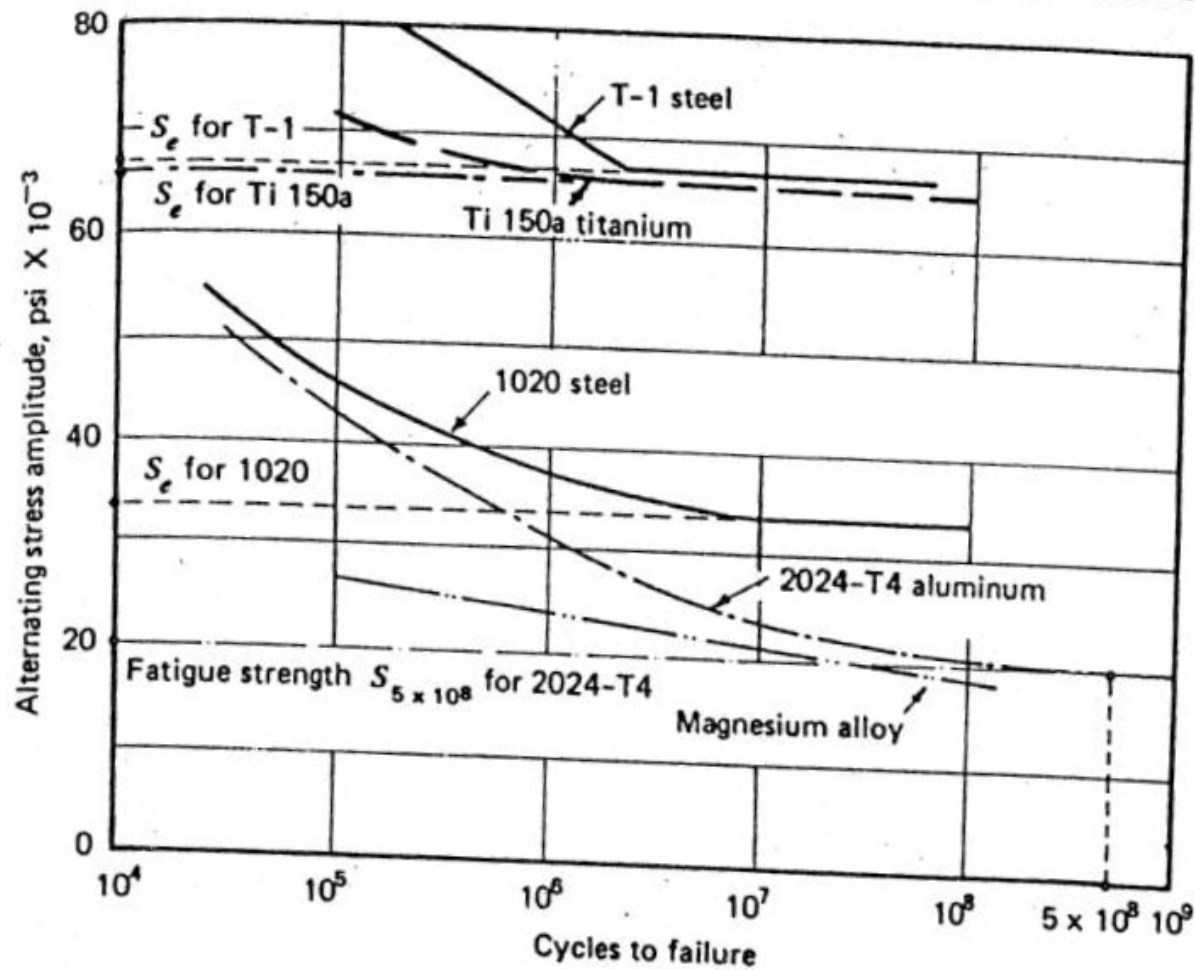
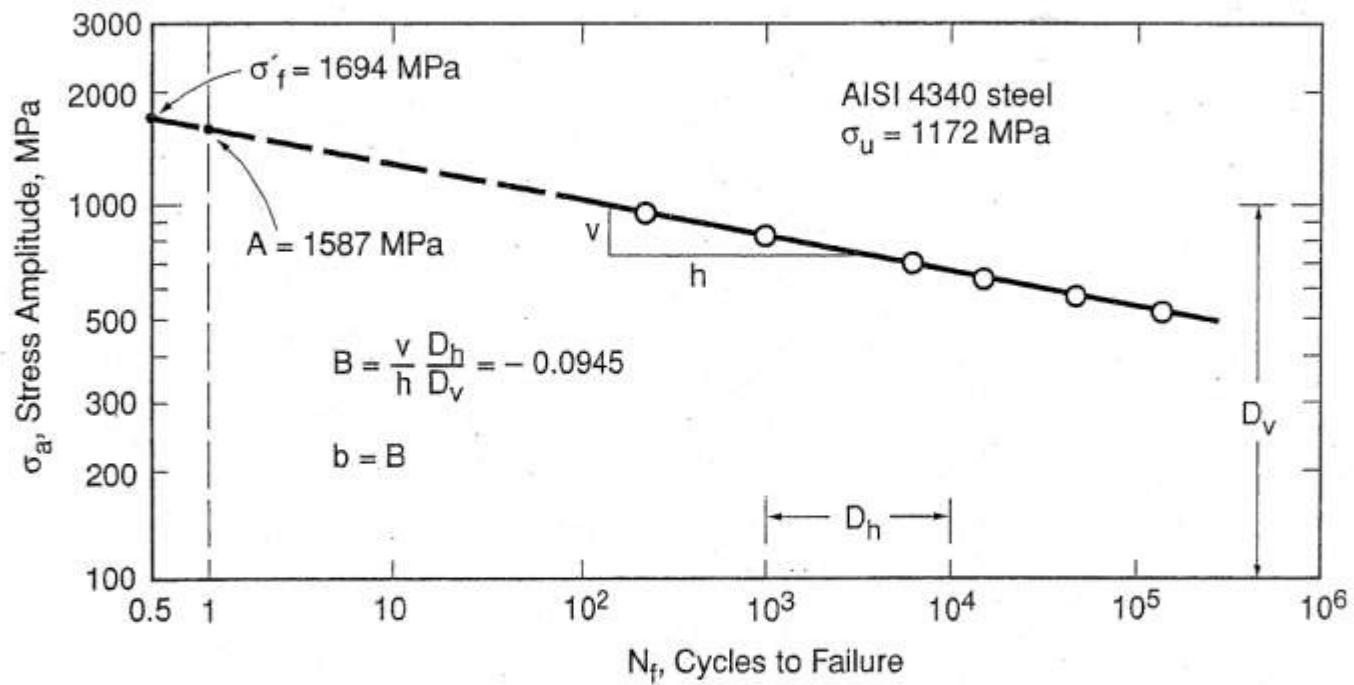


FIG. 3 Specimens with Tangentially Blending Fillets Between the Uniform Test Section and the Ends

# FACTORS THAT AFFECT $S-N-P$ CURVES





Stress-Life curve in log-log format



# S-N Data, Log-Log fit - N

$$\sigma_a = A(N_f)^B$$

- A, B are constants
- Examples:
  - 4340 steel; A= 238 ksi, B = - 0.0977
  - 2024-T4 Al; A = 122 ksi, B = - 0.102

For  $N_f = 1 \times 10^6$ ,  $\sigma_a = 122(1 \times 10^6)^{-0.102} = 29.8$  ksi

For  $\sigma_a = 40$  ksi,  $N_f = (10/122)^{-1/0.102} = 55,980$

# Variables in S-N fatigue

- Endurance limit
- Mean stress,  $\sigma_m$
- Notch effect
- Random loading

# Variables in S-N fatigue

- Endurance limit
  - For some metals, steel and Ti alloys, life is infinite below a given  $\sigma_a$
  - This is called Endurance limit,  $\sigma_E$
- Mean stress,  $\sigma_m$
- Notch effect
- Random loading

## HIGH-CYCLE FATIGUE

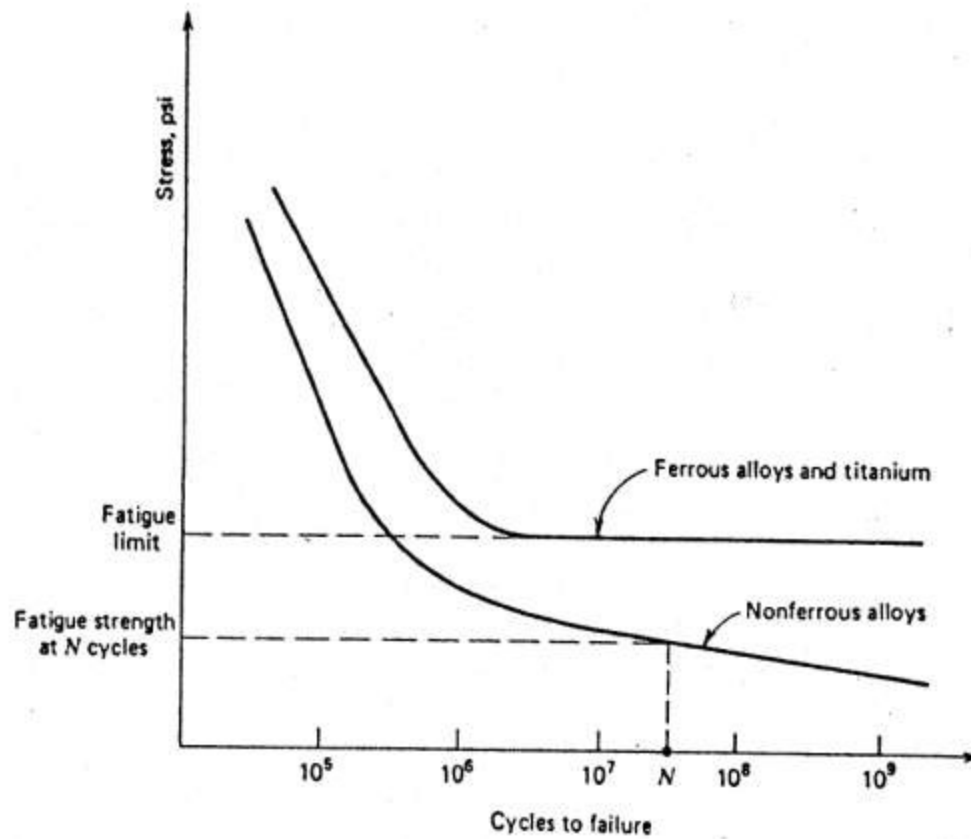
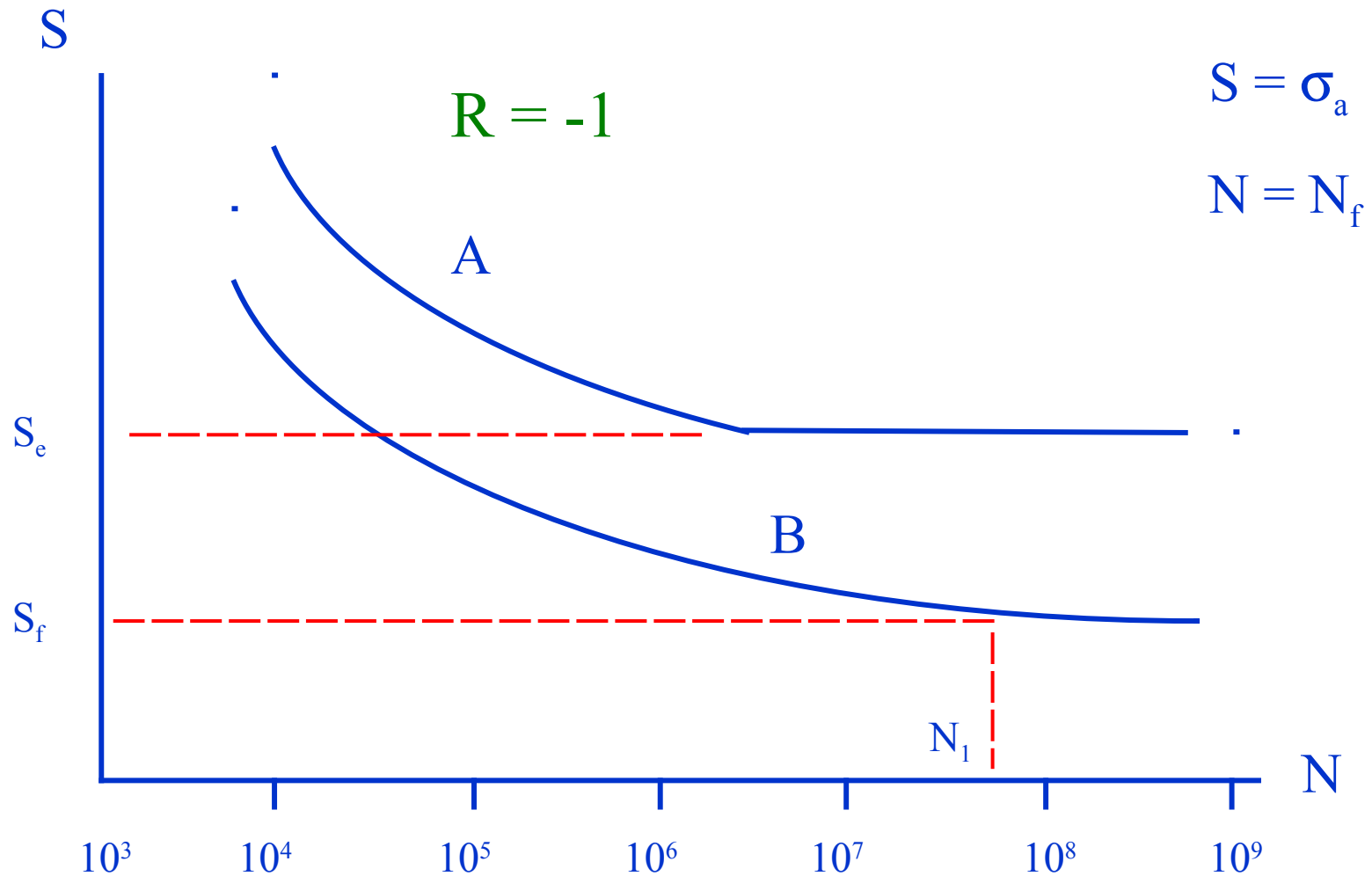


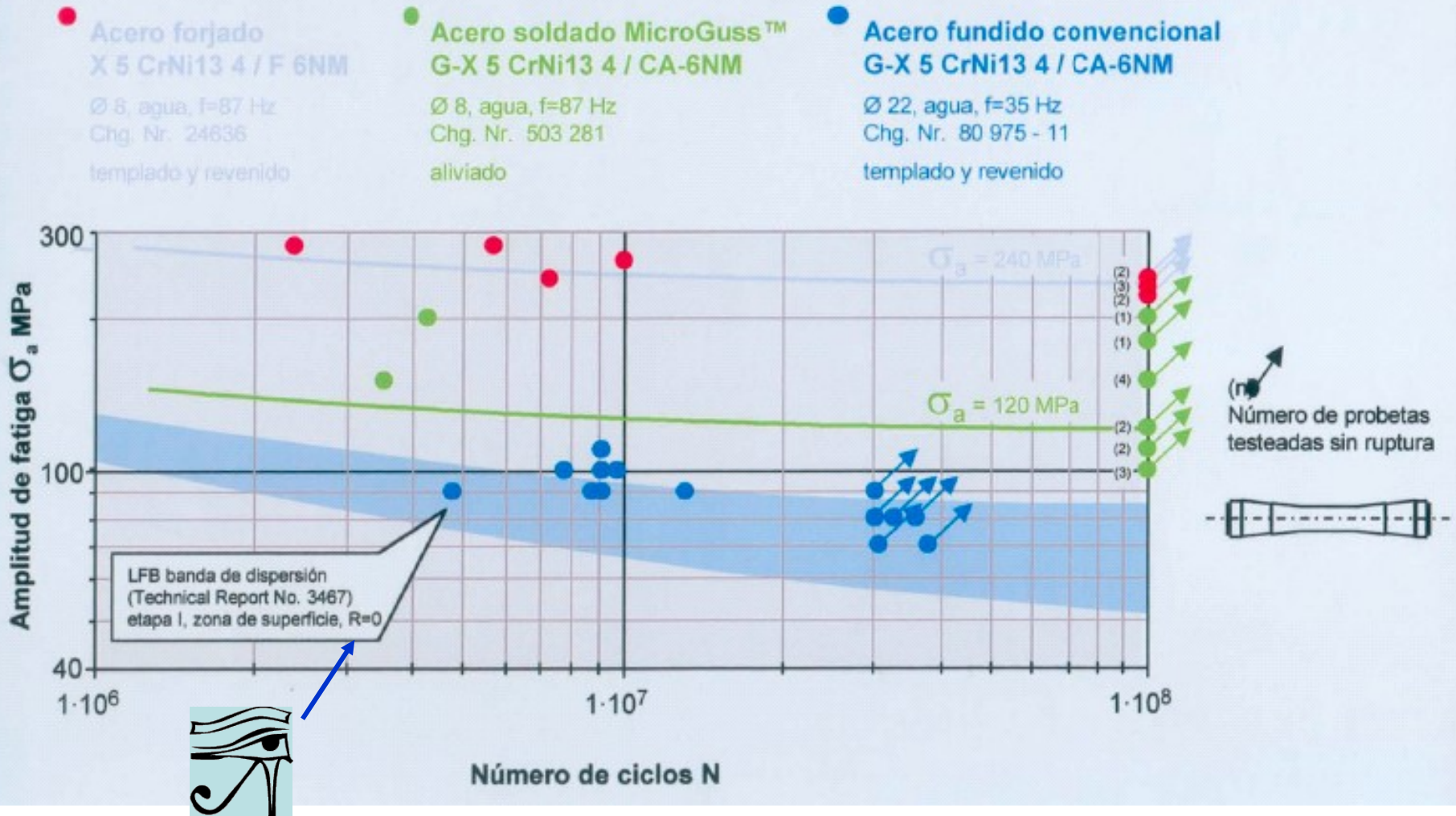
FIGURE 7.16. Two types of material response to cyclic loading.

# S-N Curve – with endurance limit



# Results from CrNi 13-4

## Comparación de resultados obtenidos de distintas pruebas de fatiga por corrosión



# Variables in S-N fatigue

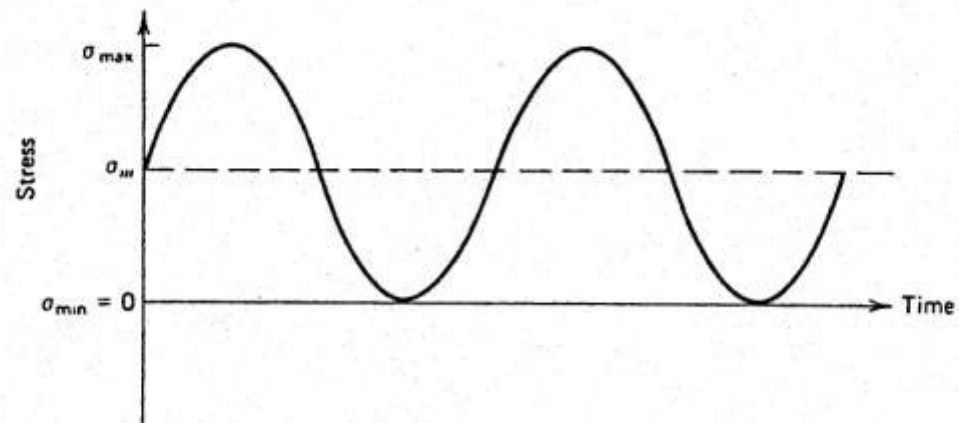
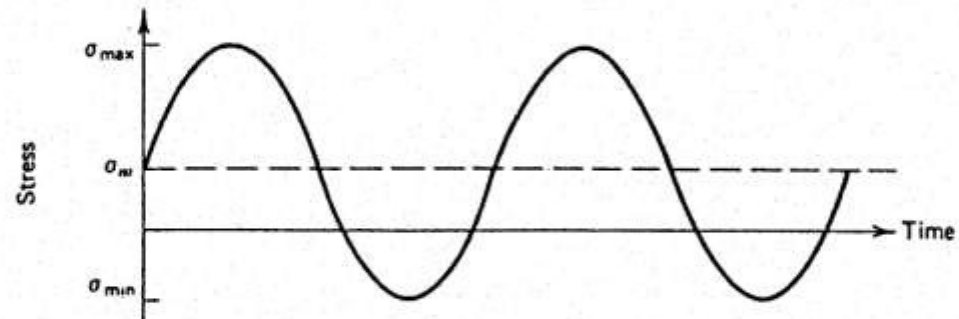
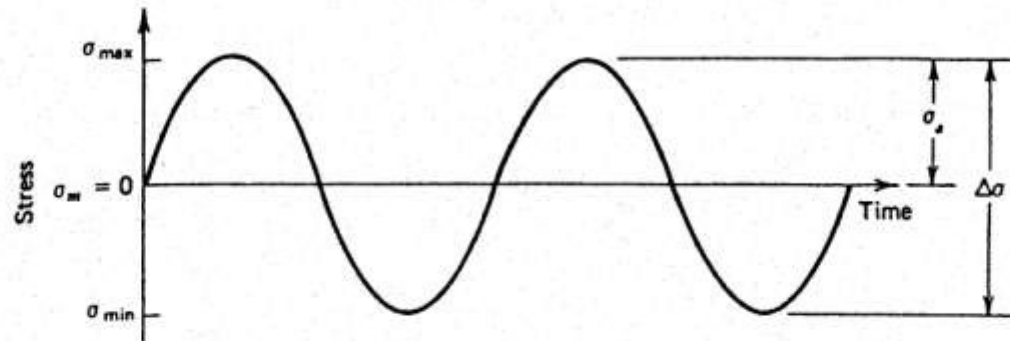
- Endurance limit
- Mean stress,  $\sigma_m$

Use Goodman Equation

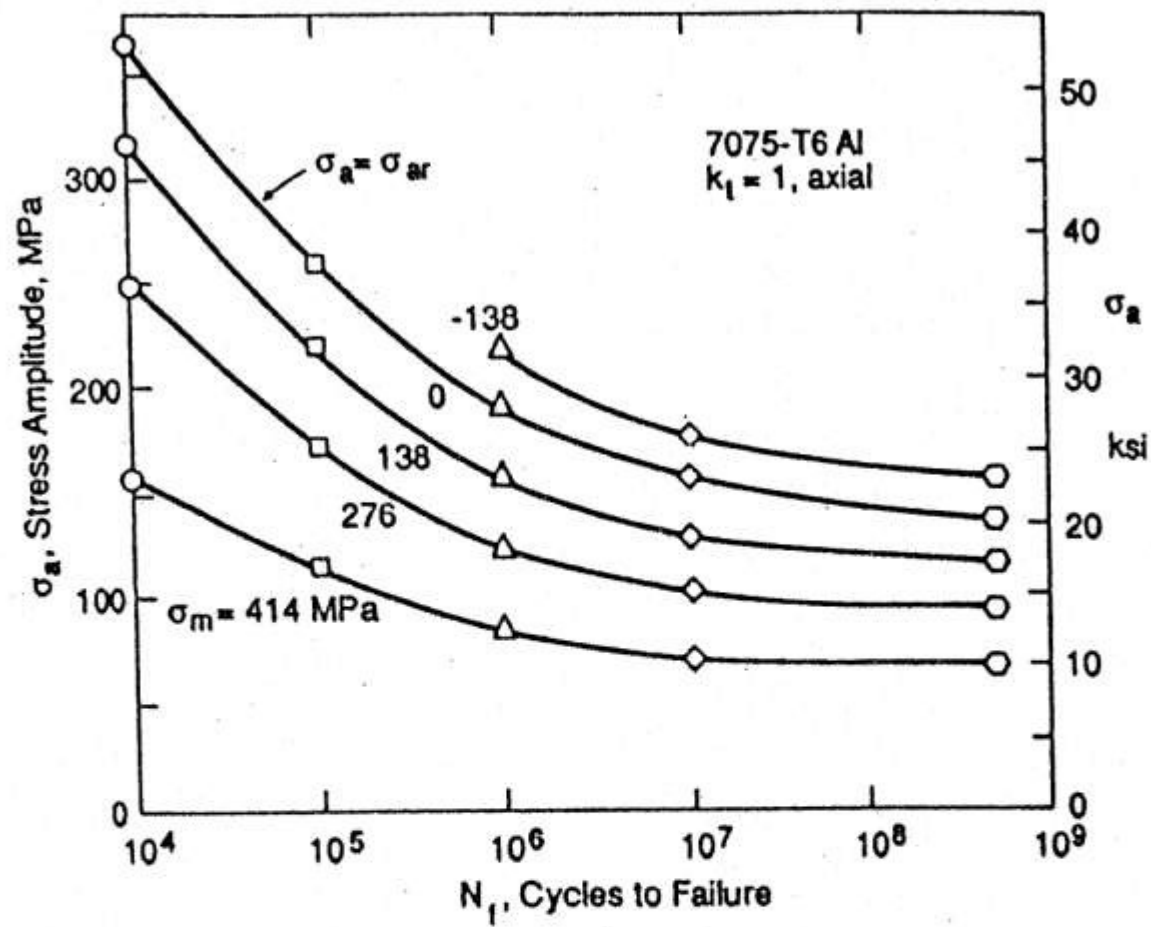
$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_{uts}} = 1$$

- Notch effect
- Random loading

# FATIGUE LOADING

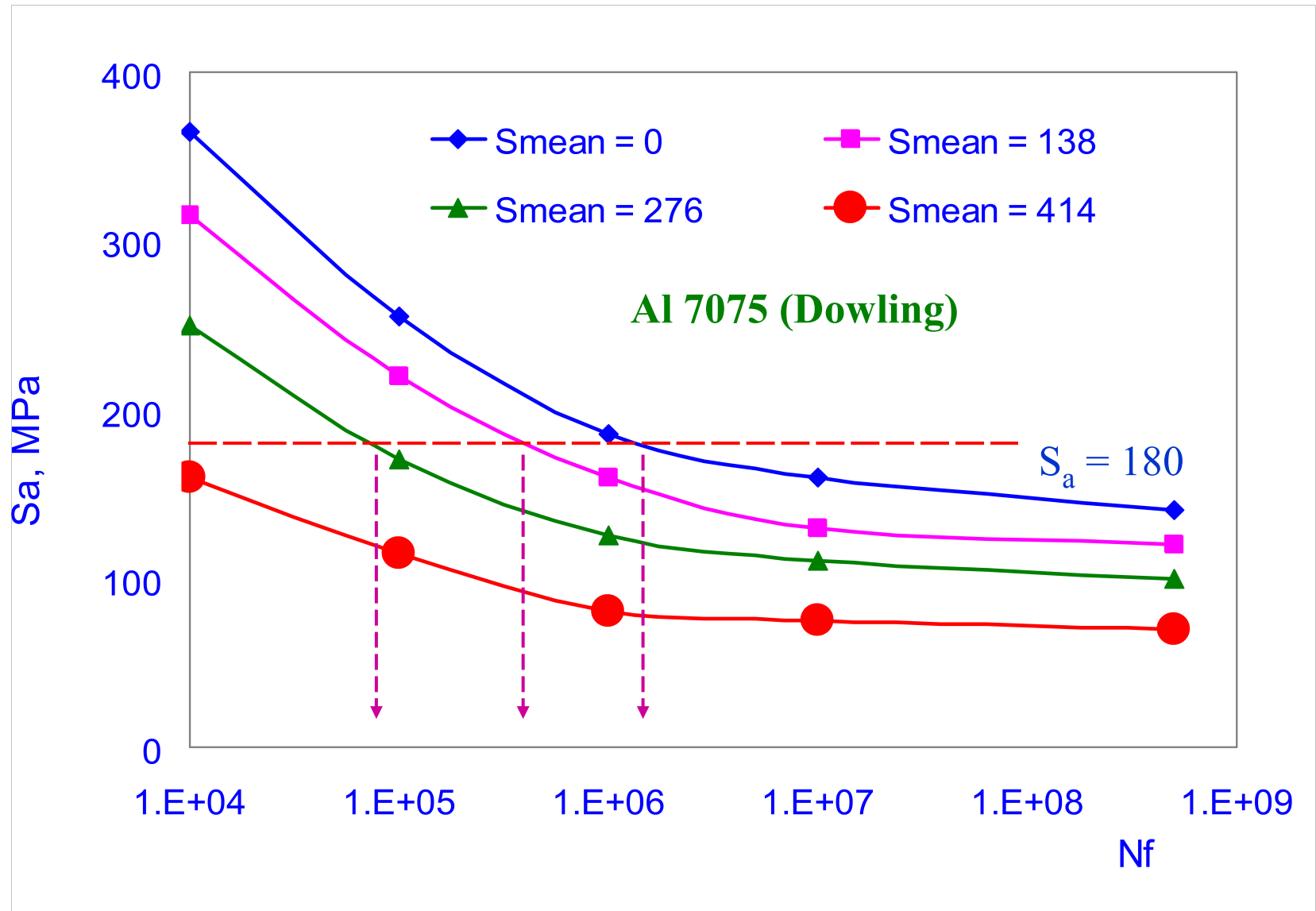


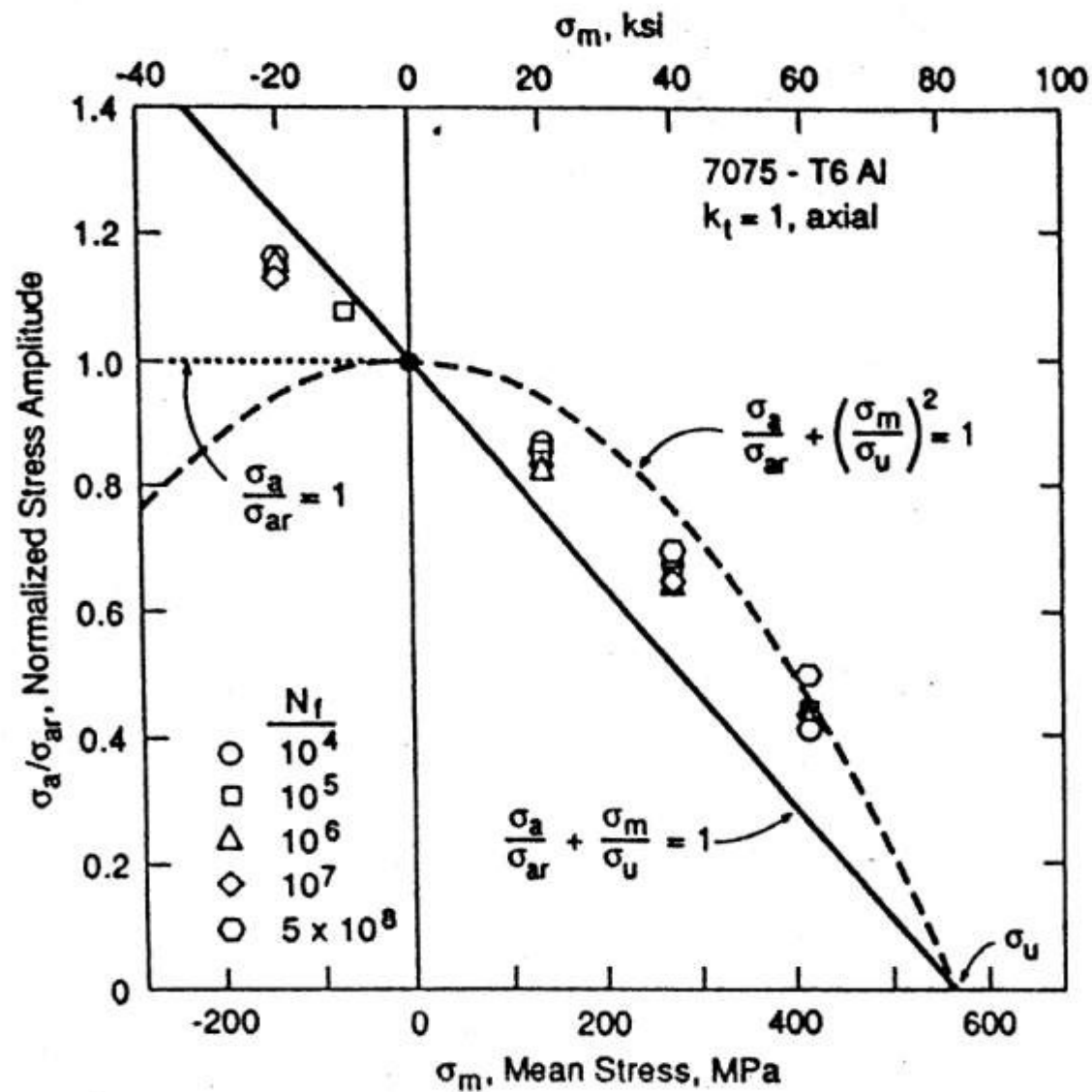




Effect of mean stress on stress-life data

# Effect of $\sigma_m$

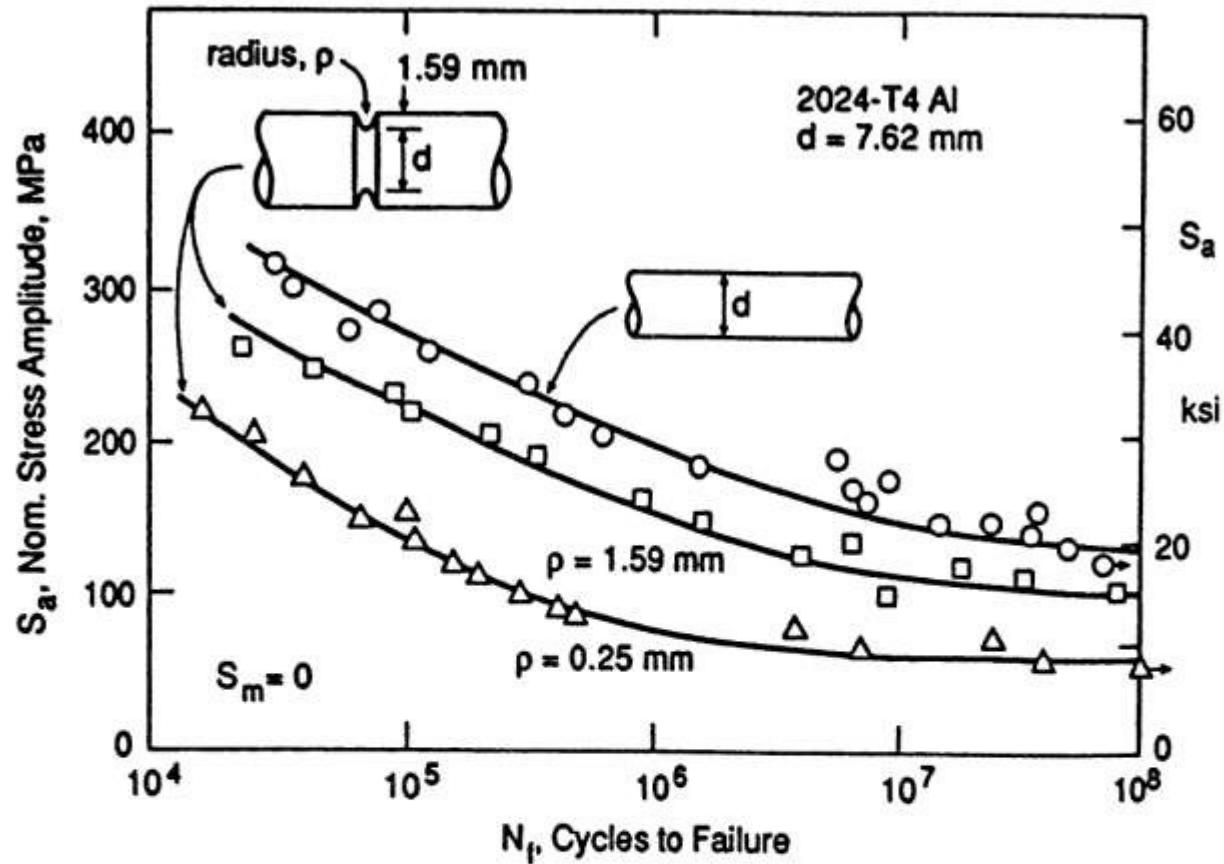




Modified Goodman Diagram

# Variables in S-N fatigue

- Endurance limit
- Mean stress,  $\sigma_m$
- Notch effect
  - Use a notch concentration factor
  - $k_t$  or  $k_f$
- Random loading



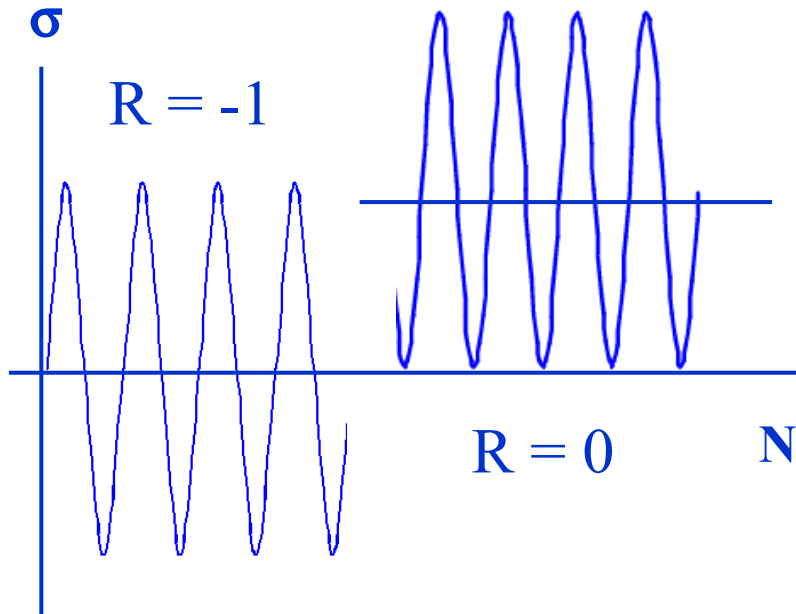
**Figure 9.26** Effects of notches having  $k_t = 1.6$  and 3.1 on rotating bending  $S$ - $N$  curves of an aluminum alloy. (Adapted from [MacGregor 52].)

# Variables in S-N fatigue

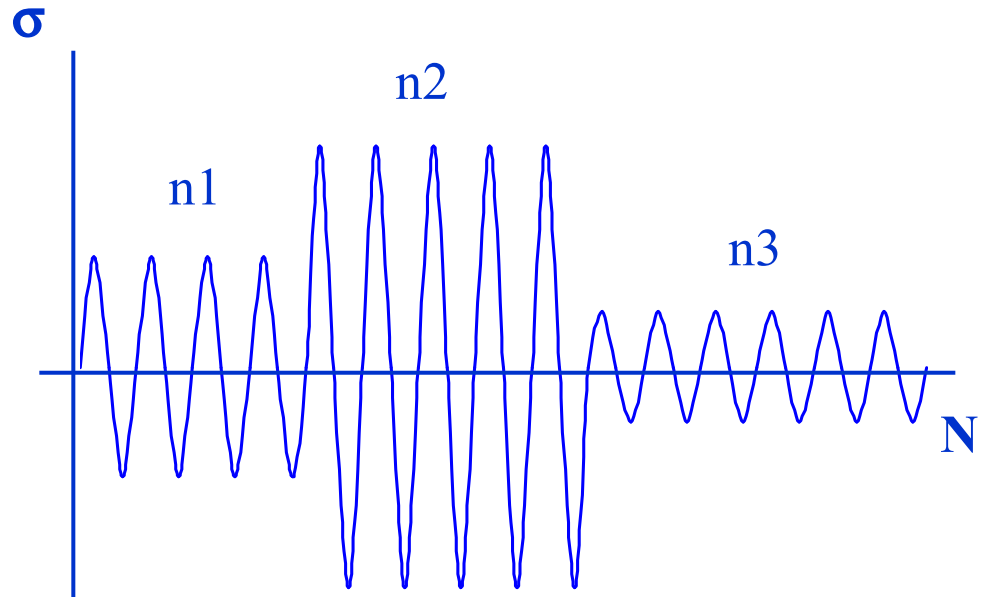
- Endurance limit
- Mean stress,  $\sigma_m$
- Notch effect
- Random loading
  - Cycle counting
  - Linear damage

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots = 1$$

# Stress range examples

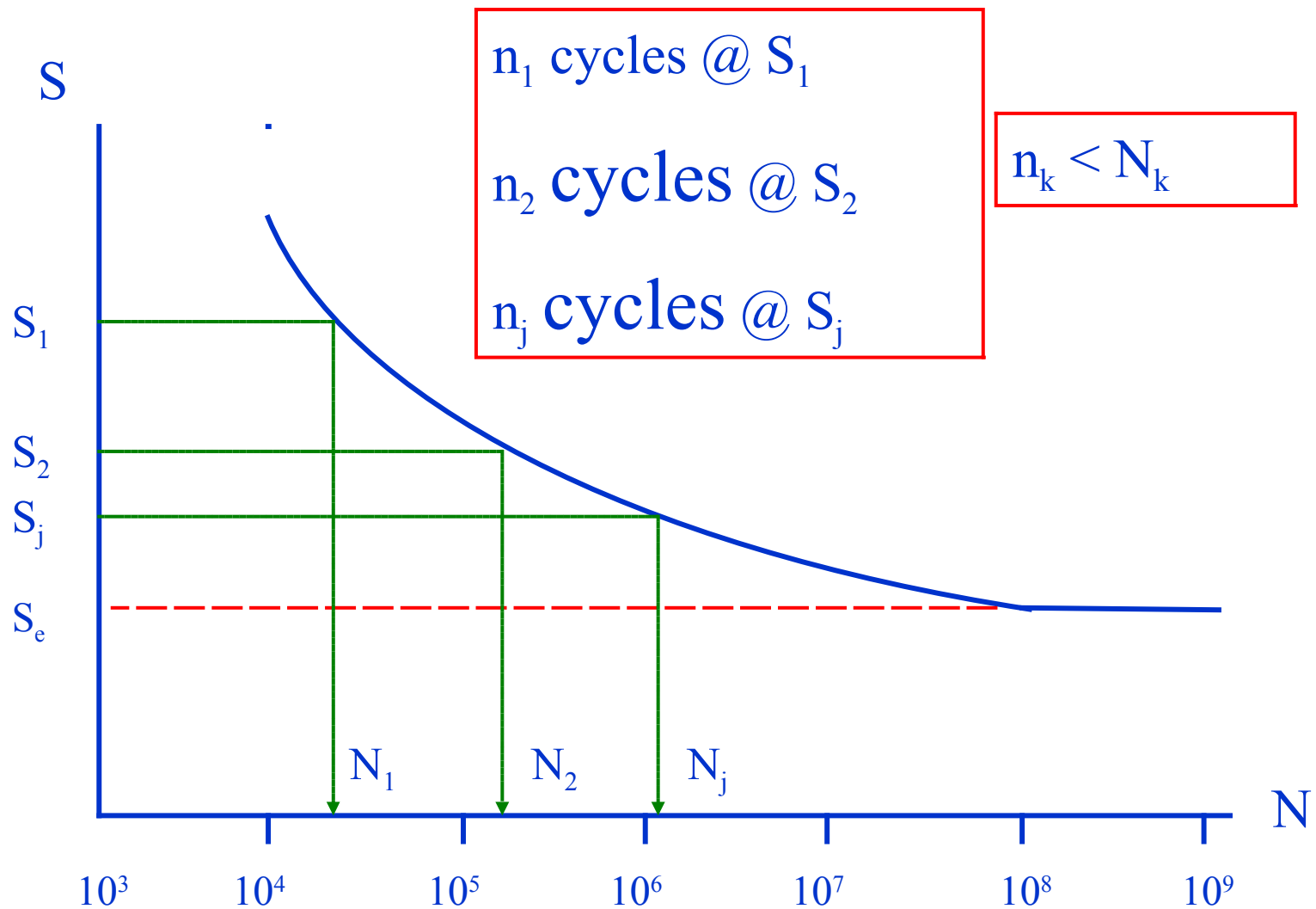


Constant amplitude,  
test



Variable amplitude,  
application

# Palmgren- Miner variables







# Low cycle fatigue life equations

- Strain amplitude is a sum of elastic and plastic strain amplitudes

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap}$$

- Fitted power law lines are combined to give total life, four fitting constants

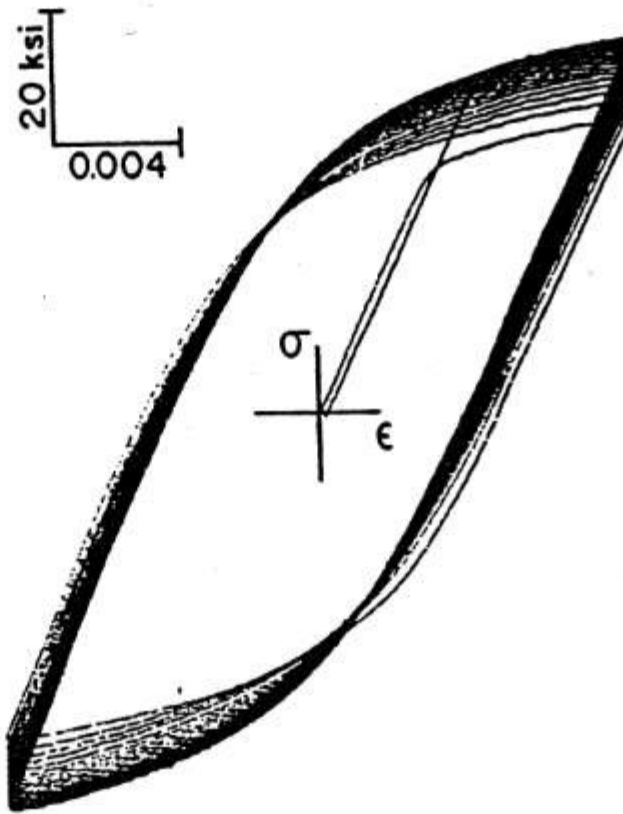
$$\varepsilon_a = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

# Low cycle fatigue, $\varepsilon$ - $N_f$

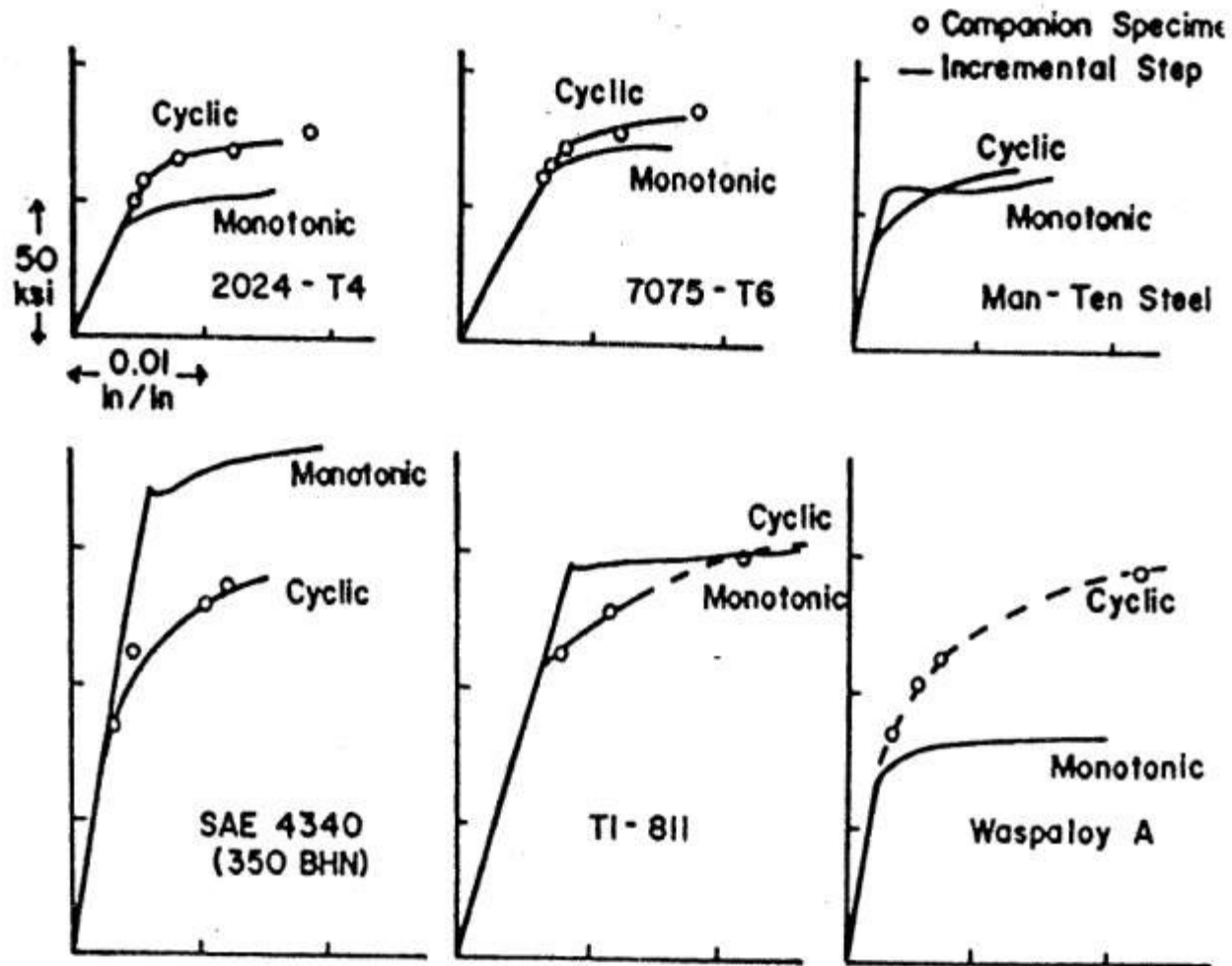
- Low cycle fatigue uses strain for the controlling parameter
- Stress and strain are related through the cyclic stress-strain curve

$$\varepsilon_a = \frac{\sigma_a}{E} + \left( \frac{\sigma_a}{H'} \right)^{\frac{1}{n'}}$$

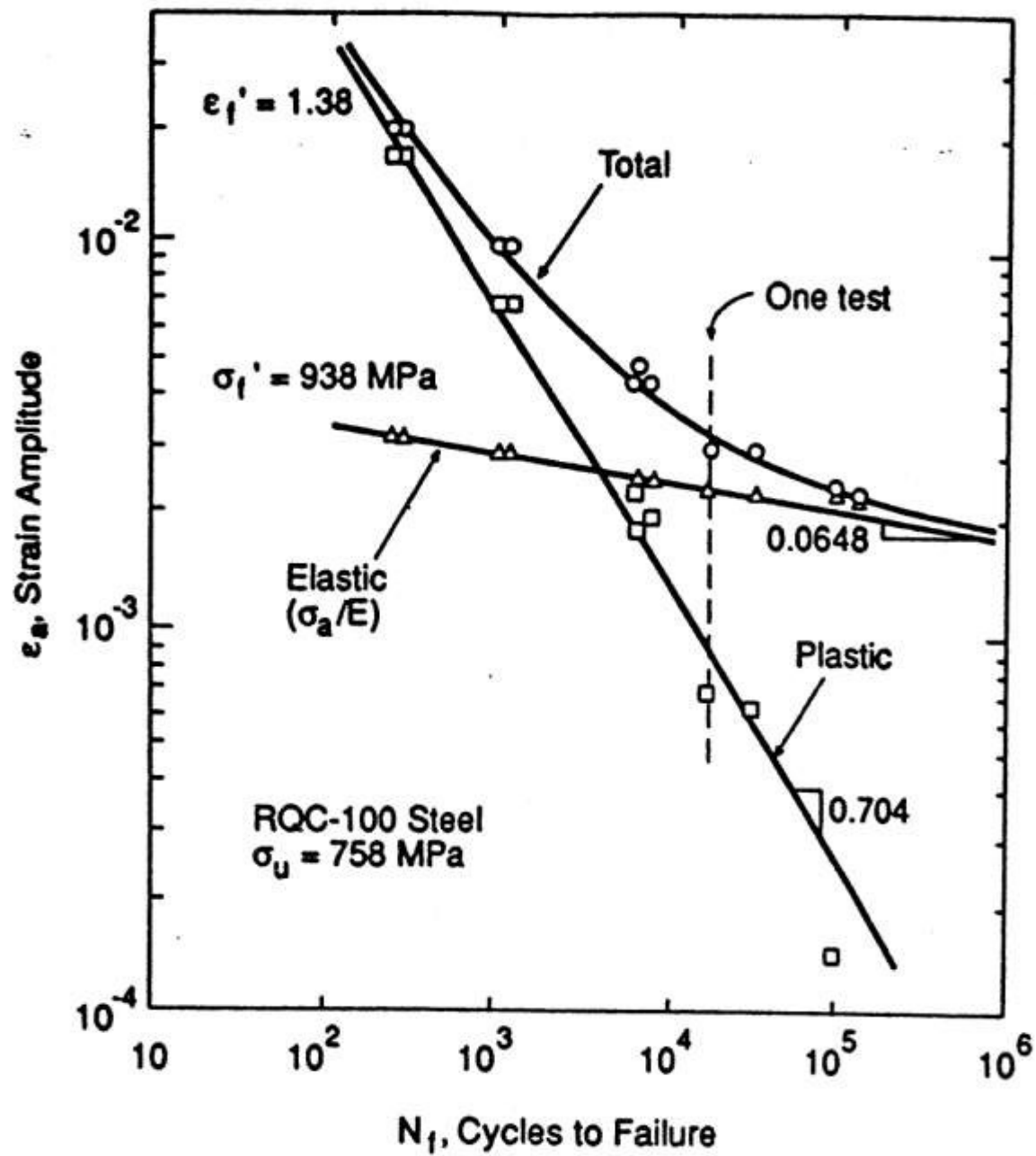
- Strain versus life is fitted with an elastic and a plastic fitting curve. These two are combined



Schematic picture of cyclic hardening  
in a 2024-T4 Aluminum Alloy



Cyclic hardening and softening stress-strain curves



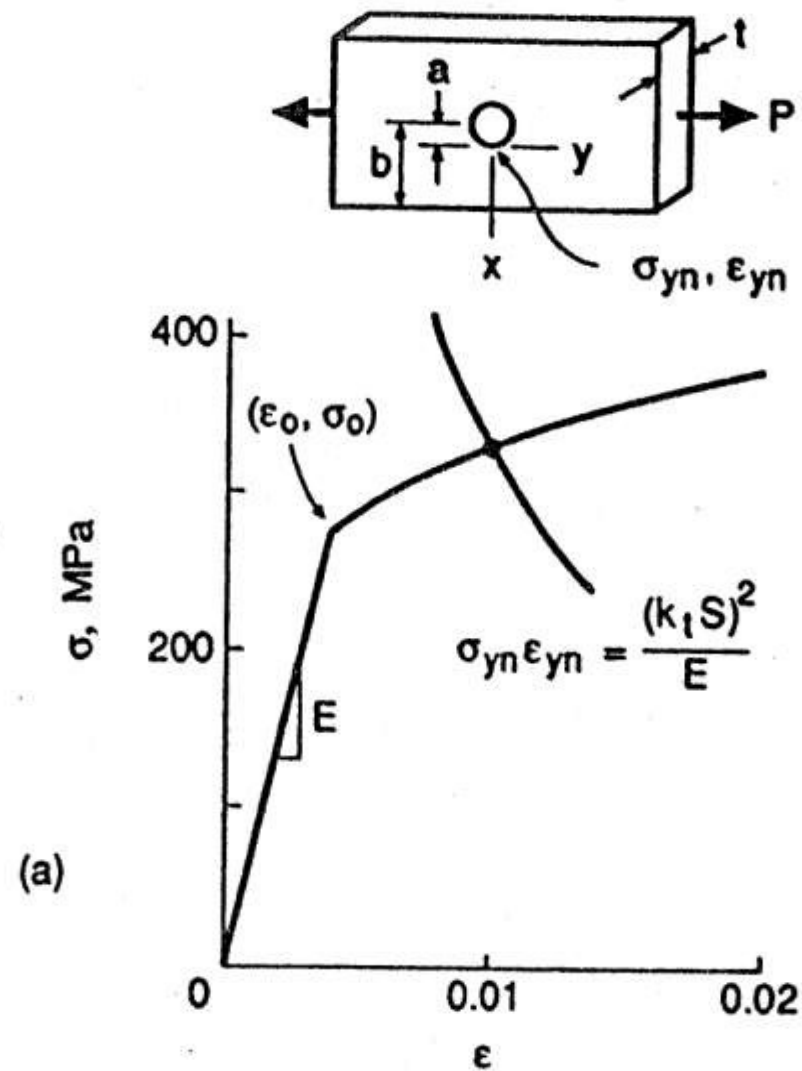
# Low cycle fatigue variables

## 1. Notch effect

- Get strain at notch tip
- Use Neuber's Rule

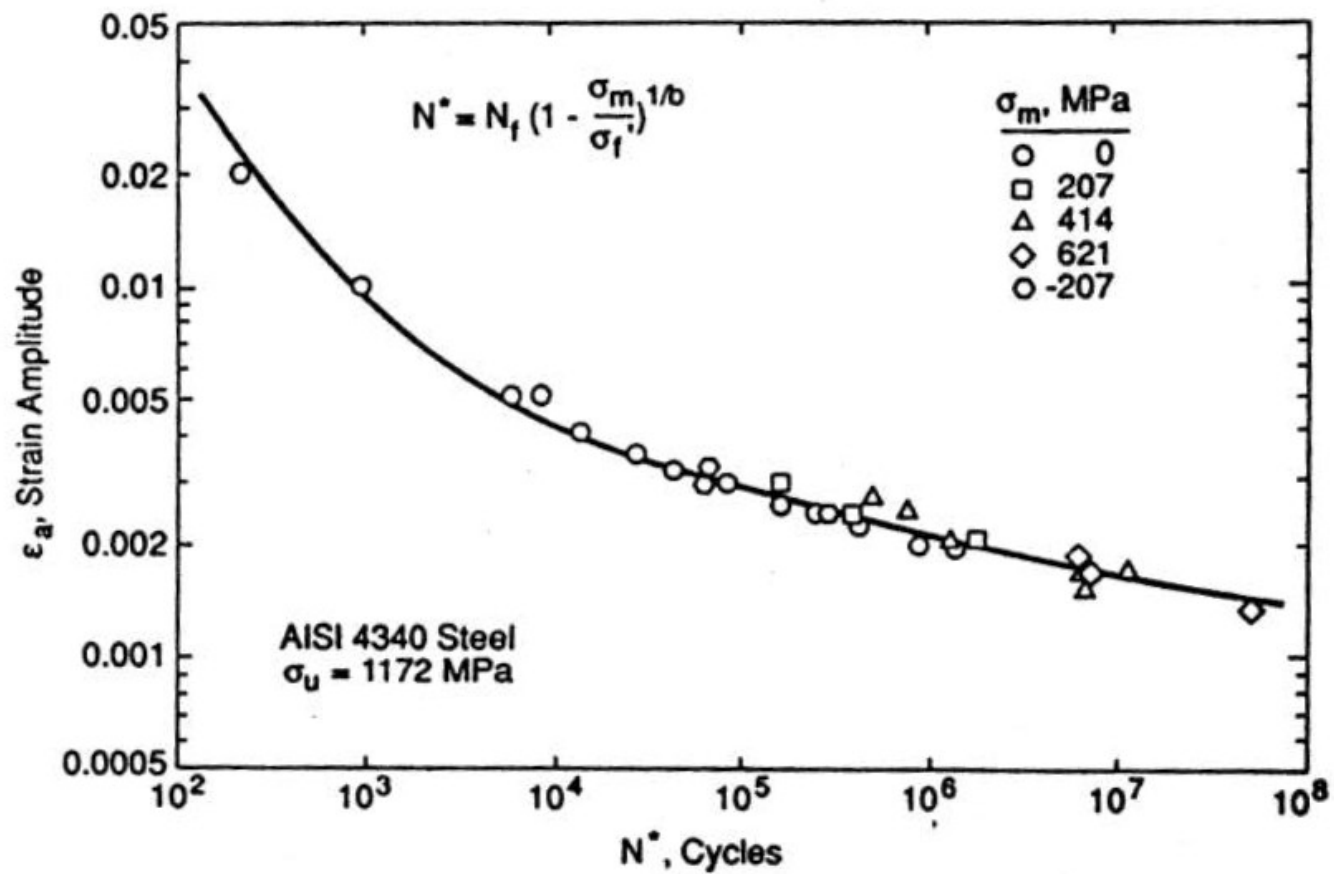
$$\sigma \times \varepsilon = \frac{(k_t S)^2}{E}$$

## 2. Mean stress effect – Morrow parameter



Neuber analysis for notched stress –strain effect



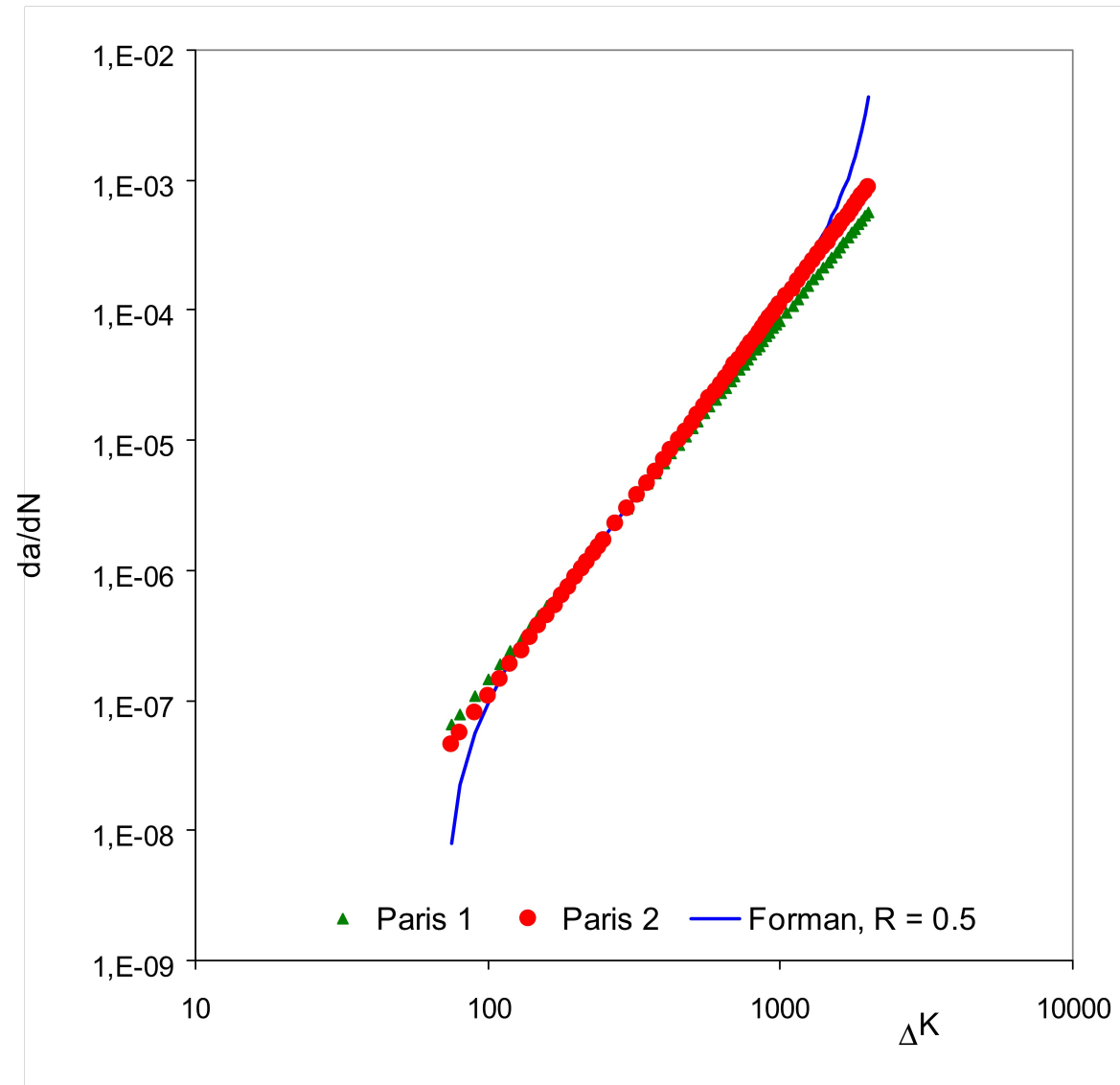


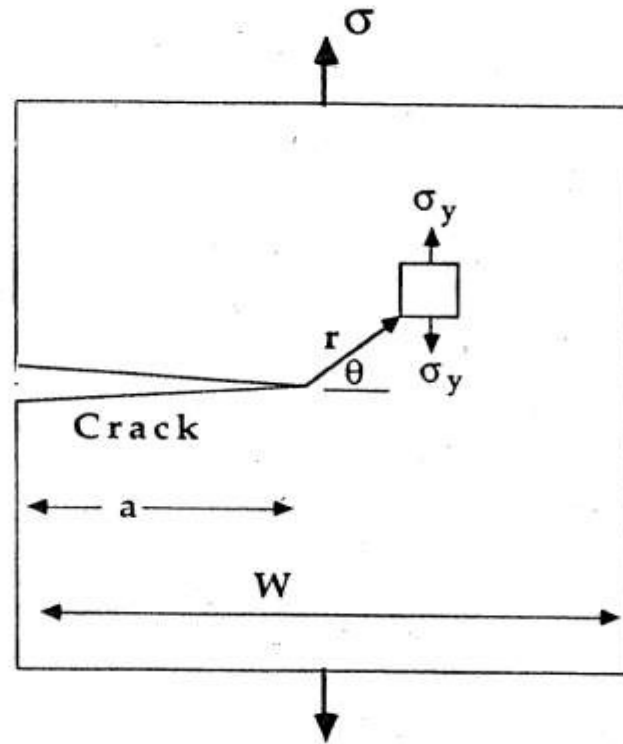
**Figure 14.11** Mean stress data of Fig. 14.10 plotted vs.  $N^*$  according to the Morrow parameter.

# Fatigue Crack Growth, $da/dN$ vs $\Delta K$

- Crack growth rate,  $da/dN$  is crack extension per fatigue cycle
- $da/dN$  is a function of the  $K$  range,  $\Delta K$
- In the middle range of crack growth rate a power law fit is used  $da/dN = C(\Delta K)^n$
- A threshold value of  $\Delta K$  is used in the low growth rate region to characterize non-growing cracks

# da/dN vs $\Delta K$ curve



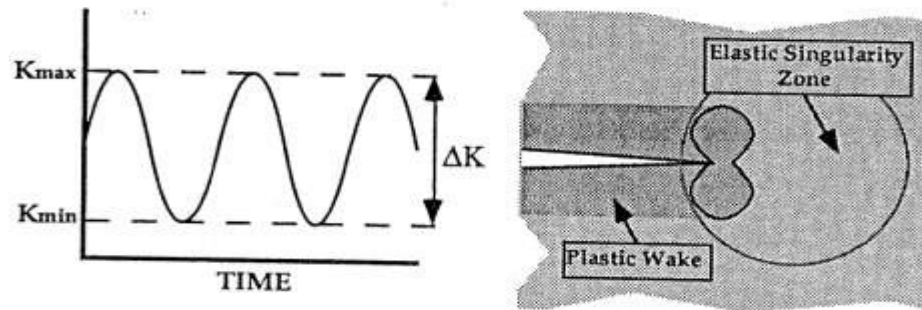


$$\sigma_y = \frac{K}{\sqrt{2\pi r}} f(\theta)$$

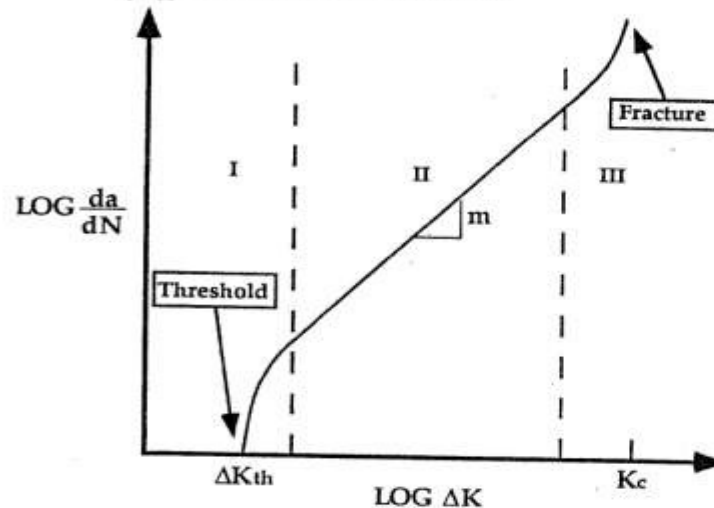
$$K = \sigma \sqrt{\pi a} f(a/W)$$

## $\Delta K$ AS A CRITERION FOR FATIGUE

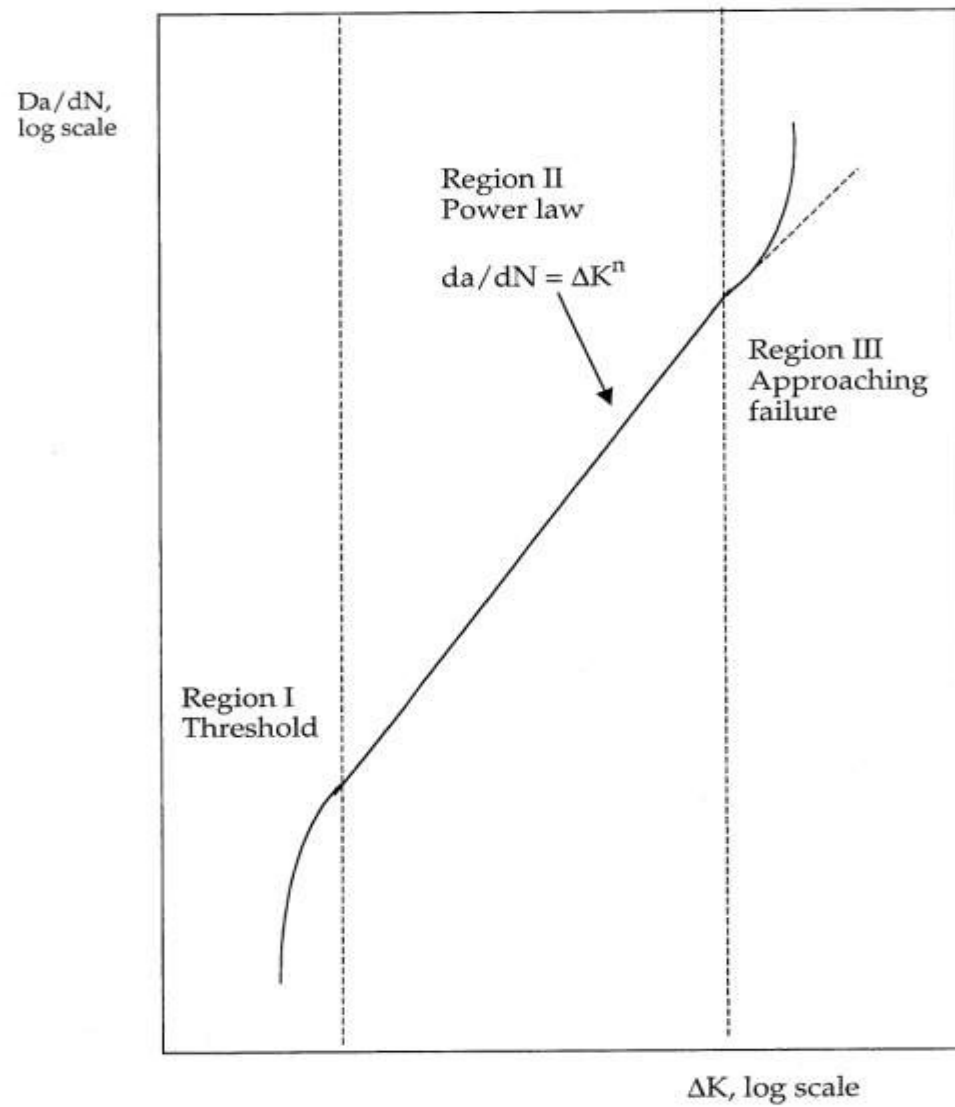
The plastic zone ahead of the propagating crack must be small in order to apply  $\Delta K$ :



### A. Typical Behavior



where  $da/dN$  is crack growth per cycle.



Schematic of Fatigue Crack Growth Rate Behavior

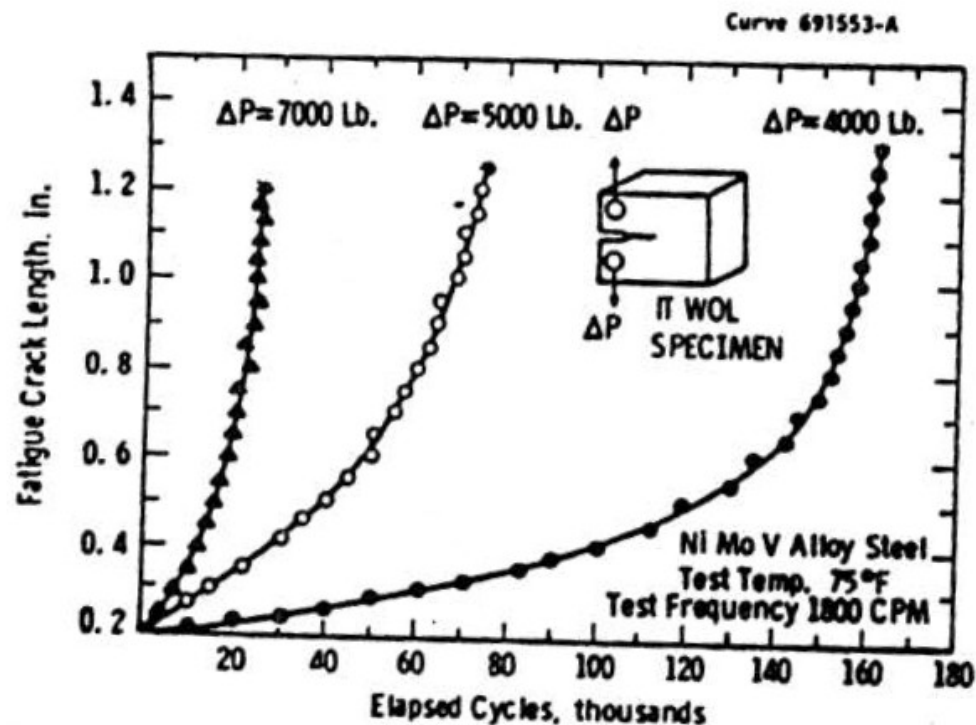


Fig. 4 - Crack growth behavior for a NiMoV steel.

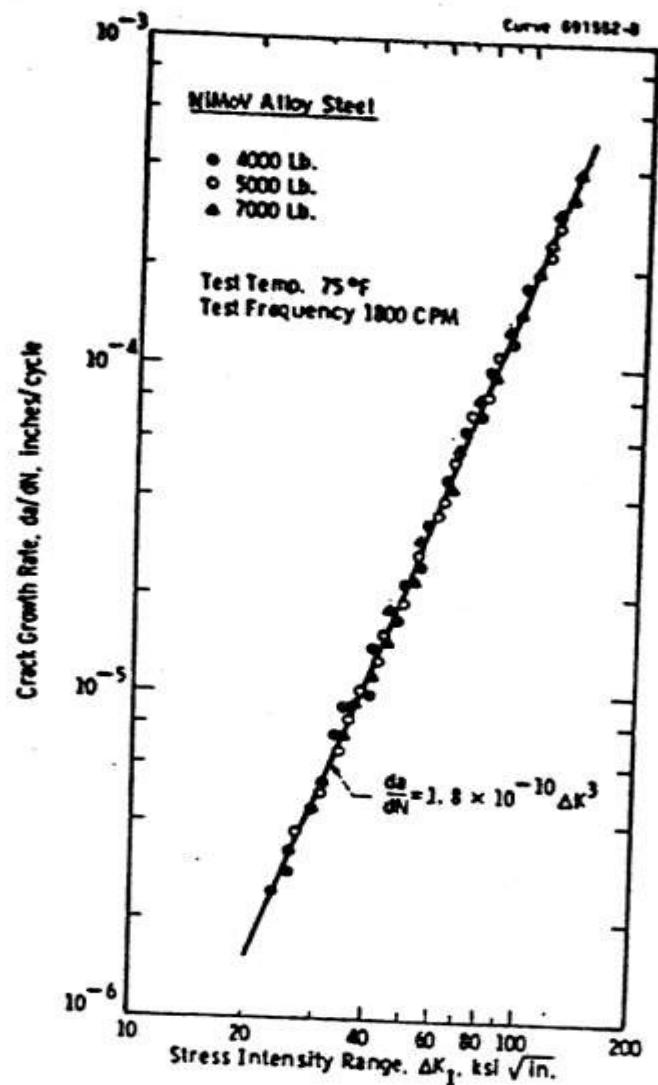


Fig. 6 - Crack growth rate behavior for a NiMoV steel.



Paris Equation for Region II:

$$\frac{da}{dN} = C \Delta K^m$$

## B. Experimental Measurement

ASTM E 647 covers the procedure for experimental determination of a  $da/dN$  -  $\Delta K$  curve.



## Standard Test Method for Measurement of Fatigue Crack Growth Rates<sup>1</sup>

This standard is issued under the fixed designation E 647; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

### 1. Scope

1.1 This test method<sup>2</sup> covers the determination of steady-state fatigue crack growth rates from near-threshold to  $K_{max}$  controlled instability using either compact tension, C(T), (Fig. 1) or middle-tension, M(T), (Fig. 2) specimens. Results are expressed in terms of the crack-tip stress-intensity factor range ( $\Delta K$ ), defined by the theory of linear elasticity.

1.2 Several different test procedures are provided, the optimum test procedure being primarily dependent on the magnitude of the fatigue crack growth rate to be measured.

1.3 Materials that can be tested by this test method are not limited by thicknesses or by strength so long as specimens are of sufficient thickness to preclude buckling and of sufficient planar size to remain predominantly elastic during testing.

1.4 A range of specimen sizes with proportional planar dimensions is provided, but size is variable to be adjusted for yield strength and applied load. Specimen thickness may be varied independent of planar size.

1.5 Specimen configurations other than those contained in this method may be used provided that well-established stress-intensity factor calibrations are available and that specimens are of sufficient planar size to remain predominantly elastic during testing.

1.6 Residual stress/crack closure may significantly influence the fatigue crack growth rate data, particularly at low stress-intensity factors and low stress ratios, although such variables are not incorporated into the computation of  $\Delta K$ .

1.7 Values stated in SI units are to be regarded as the standard. Values given in parentheses are for information only.

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

### 2. Referenced Documents

#### 2.1 ASTM Standards:

- E 4 Practices for Force Verification of Testing Machines<sup>3</sup>
- E 6 Terminology Relating to Methods of Mechanical Testing<sup>3</sup>
- E 8 Test Methods for Tension Testing of Metallic Materials<sup>3</sup>

<sup>1</sup> This test method is under the jurisdiction of ASTM Committee E-8 on Fatigue and Fracture and is the direct responsibility of Subcommittee E08.06 on Crack Growth Behavior.

Current edition approved Oct. 10, 1995. Published December 1995. Originally published as E 647 - 78 T. Last previous edition E 647 - 95.

<sup>2</sup> For additional information on this test method see RR: E 24 - 1001. Available from ASTM Headquarters, 1916 Race Street, Philadelphia, PA 19103.

<sup>3</sup> Annual Book of ASTM Standards, Vol 03.01.

E 337 Test Method for Measuring Humidity with a Psychrometer (the Measurement of Wet- and Dry-Bulb Temperatures)<sup>4</sup>

E 338 Test Method for Sharp-Notch Tension Testing of High-Strength Sheet Materials<sup>3</sup>

E 399 Test Method for Plane-Strain Fracture Toughness of Metallic Materials<sup>3</sup>

E 467 Practice for Verification of Constant Amplitude Dynamic Loads on Displacements in an Axial Load Fatigue Testing System<sup>3</sup>

E 561 Practice for R-Curve Determination<sup>3</sup>

E 616 Terminology Relating to Fracture Testing<sup>3</sup>

E 813 Test Method for  $J_{10}$ , A Measure of Fracture Toughness<sup>3</sup>

E 1012 Practice for Verification of Specimen Alignment Under Tensile Loading<sup>3</sup>

E 1150 Definitions of Terms Relating to Fatigue<sup>3</sup>

### 3. Terminology

3.1 The terms used in this test method are given in Terminology E 6, Definitions E 1150, and Terminology E 616. Wherever these terms are not in agreement with one another, use the definitions given in Terminology E 616 which are applicable to this test method.

#### 3.2 Definitions:

3.2.1 *crack length*,  $a[L]$ ,  $n$ —See *crack size*.

3.2.2 *crack size*,  $a[L]$ ,  $n$ —A linear measure of a principal planar dimension of a crack. This measure is commonly used in the calculation of quantities descriptive of the stress and displacement fields and is often also termed crack length or depth.

3.2.2.1 *Discussion*—In the C(T) specimen,  $a$  is measured from the line connecting the bearing points of load application; in the M(T) specimen,  $a$  is measured from the perpendicular bisector of the central crack.

3.2.2.2 *Discussion*—In fatigue testing, crack length is the physical crack size. See *physical crack size* in Terminology E 616.

3.2.3 *cycle*—in fatigue, under constant amplitude loading, the load variation from the minimum to the maximum and then to the minimum load.

3.2.3.1 *Discussion*—In spectrum loading, the definition of cycle varies with the counting method used.

3.2.3.2 *Discussion*—In this test method, the symbol  $N$  is used to represent the number of cycles.

3.2.4 *fatigue-crack-growth rate*,  $da/dN$ ,  $[L]$ —crack extension per cycle of loading.

3.2.5 *fatigue cycle*—See *cycle*.

<sup>4</sup> Annual Book of ASTM Standards, Vol 11.03.

# Latest Crack Growth Rate Standard

- **ASTM E647 - 11 Standard Test Method for Measurement of Fatigue Crack Growth Rates**

## E 647 da/dN vs $\Delta K$ Standard requirements

1. Specimens: C(T), M(T)
2. Cycle at constant  $\Delta P$ , measure  $a$  versus  $N$
3. Measure Crack length using
  - Visual
  - Compliance
  - Electrical potential
4. Analyze crack length by
  - Secant
  - Incremental polynomial
5. Size requirements
  - C(T);  $(W - a) \geq \frac{4}{\pi} \left( \frac{K}{\sigma_{ys}} \right)^2$
  - M(T);  $(2W - 2a) \geq \frac{1.25 P_{\max}}{B \sigma_{ys}} \quad (\sigma_{\text{net}} \leq 0.8 \sigma_{ys})$

# Methods for measuring crack length

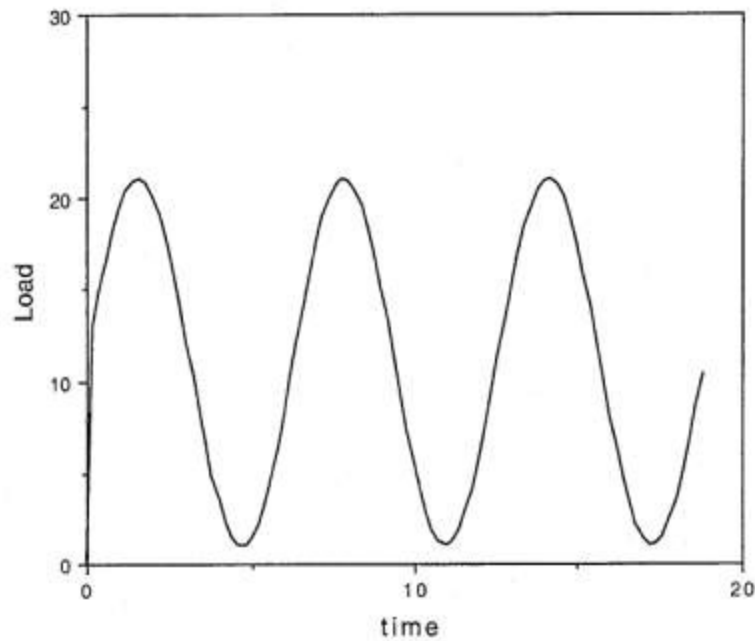
1. Visual – traveling microscope, lines on a specimen, calibrated rule, photographic
2. Compliance – load versus displacement slopes
3. Electrical potential drop – electrodes directly on the specimen; AC and DC
4. Other – ultrasonics, crack gage

# Visual crack length measurement

1. Use a microscope with a calibrated traveling stage to measure crack length,  $a$ , versus elapsed cycles,  $N$  as the crack grows
2. Crack length,  $a$ , is a surface measurement, usually only on one side
3. Alternate, put calibrated lines on the specimen surface and view with a magnifying glass

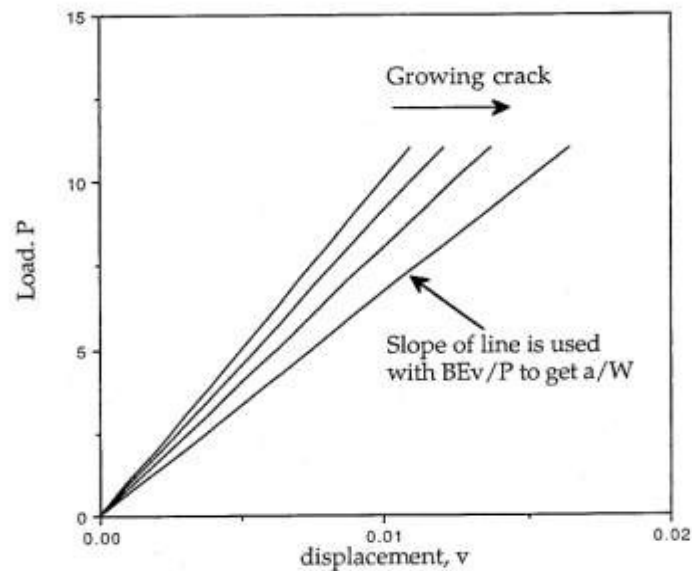
# Compliance measurement of crack length

1. Given a constant amplitude cyclic load pattern



**Constant amplitude load cycles**

2. Measure the slopes of the load versus displacement curve



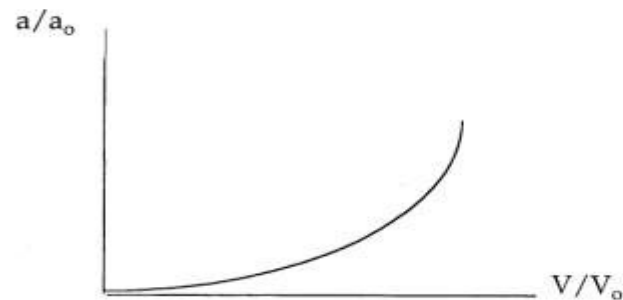
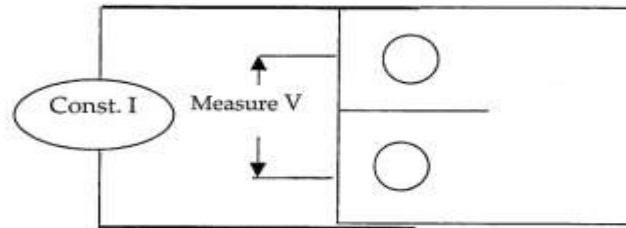
Load versus displacement for a cyclic load test

3. Use  $BEv/P$  with calibration to get  $a/W$  and crack length  $a$



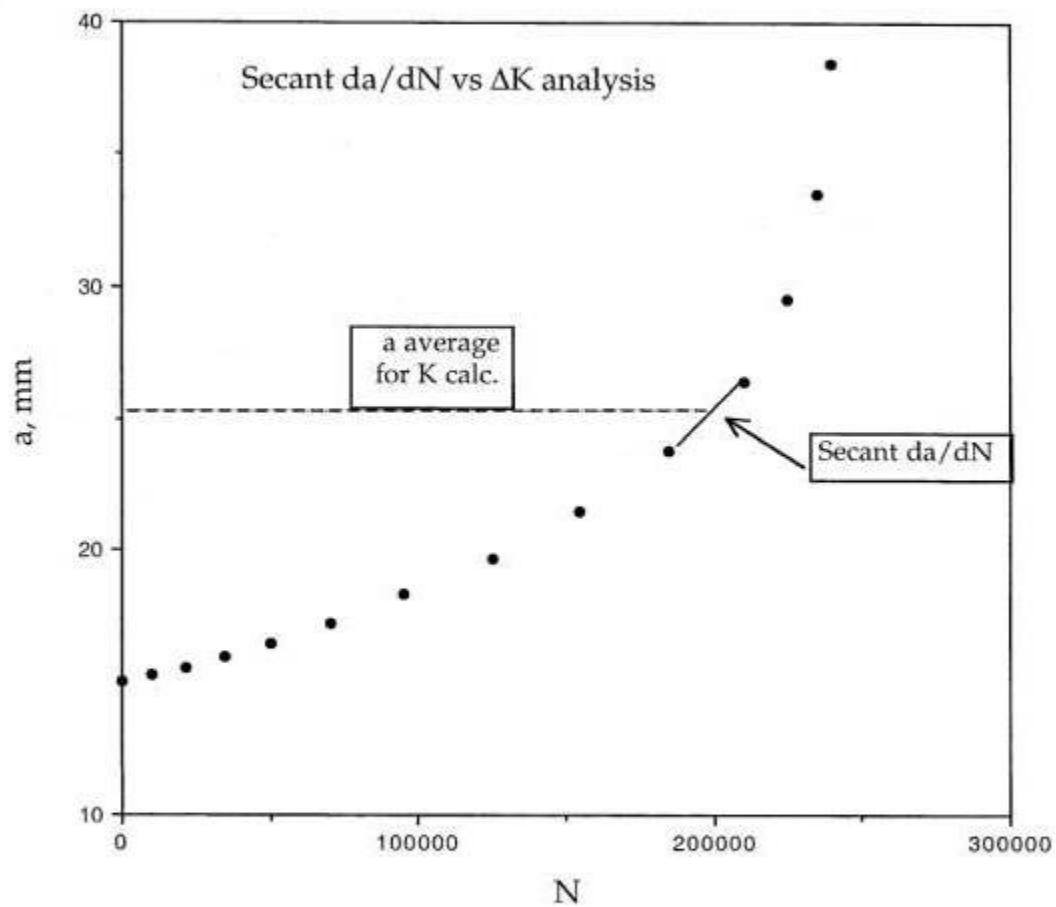
# Potential Drop Crack length measurement

1. Put a constant electrical current through the specimen
2. The electrical resistance changes with decreasing uncracked area
3. A voltage measured at a calibrated point, gives the change in resistance;  $V = IR$
4. Plot  $a/a_0$  versus  $V/V_0$

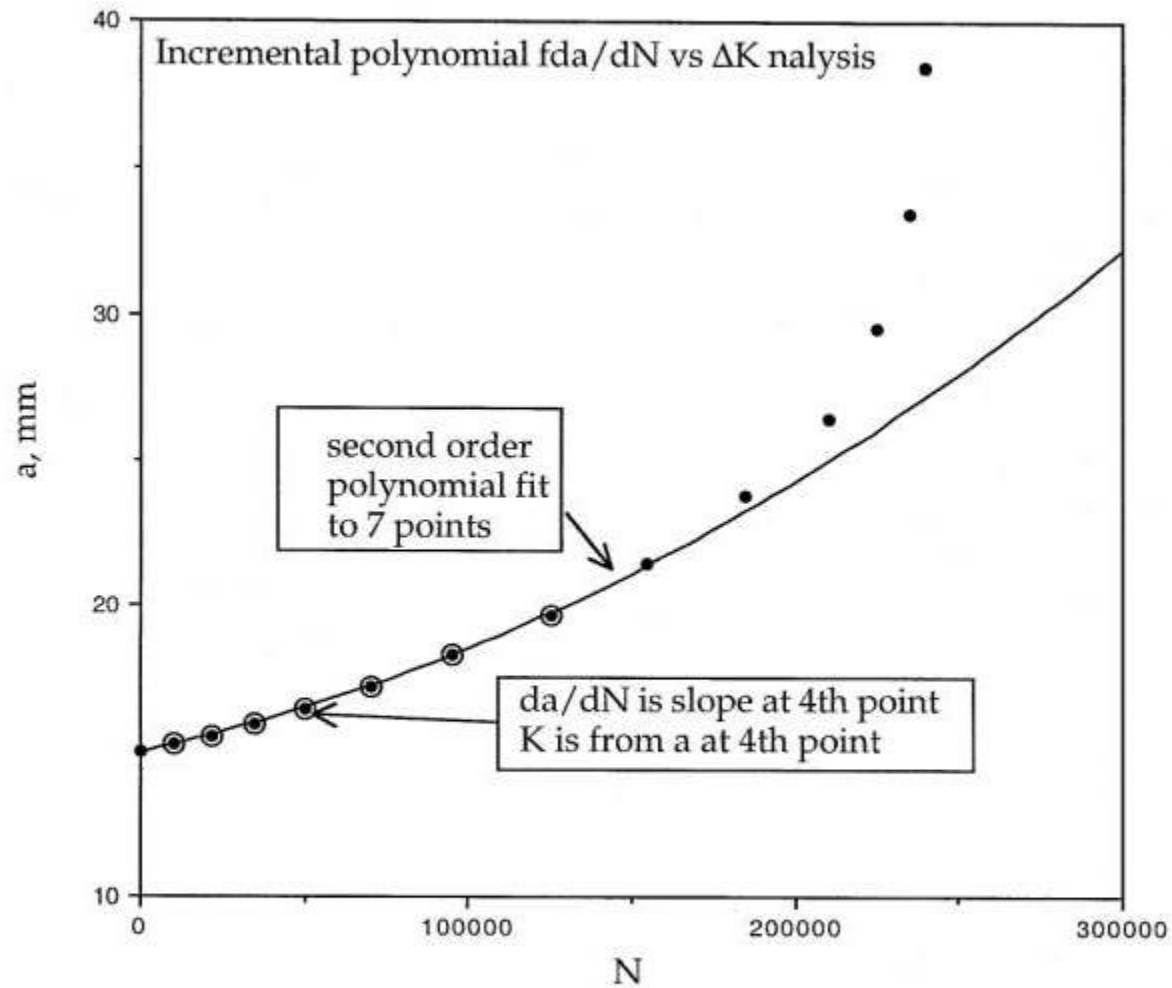


## E 647 da/dN vs $\Delta K$ Standard requirements

1. Specimens: C(T), M(T)
2. Cycle at constant  $\Delta P$ , measure  $a$  versus  $N$
3. Measure Crack length using
  - Visual
  - Compliance
  - Electrical potential
4. Analyze crack length by
  - Secant
  - Incremental polynomial
5. Size requirements
  - C(T);  $(W - a) \geq \frac{4}{\pi} \left( \frac{K}{\sigma_{ys}} \right)^2$
  - M(T);  $(2W - 2a) \geq \frac{1.25 P_{\max}}{B \sigma_{ys}} \quad (\sigma_{\text{net}} \leq 0.8 \sigma_{ys})$



$a$  versus  $N$  from fatigue crack growth



a versus N from fatigue crack growth

# Lecture 6

## Fatigue Properties

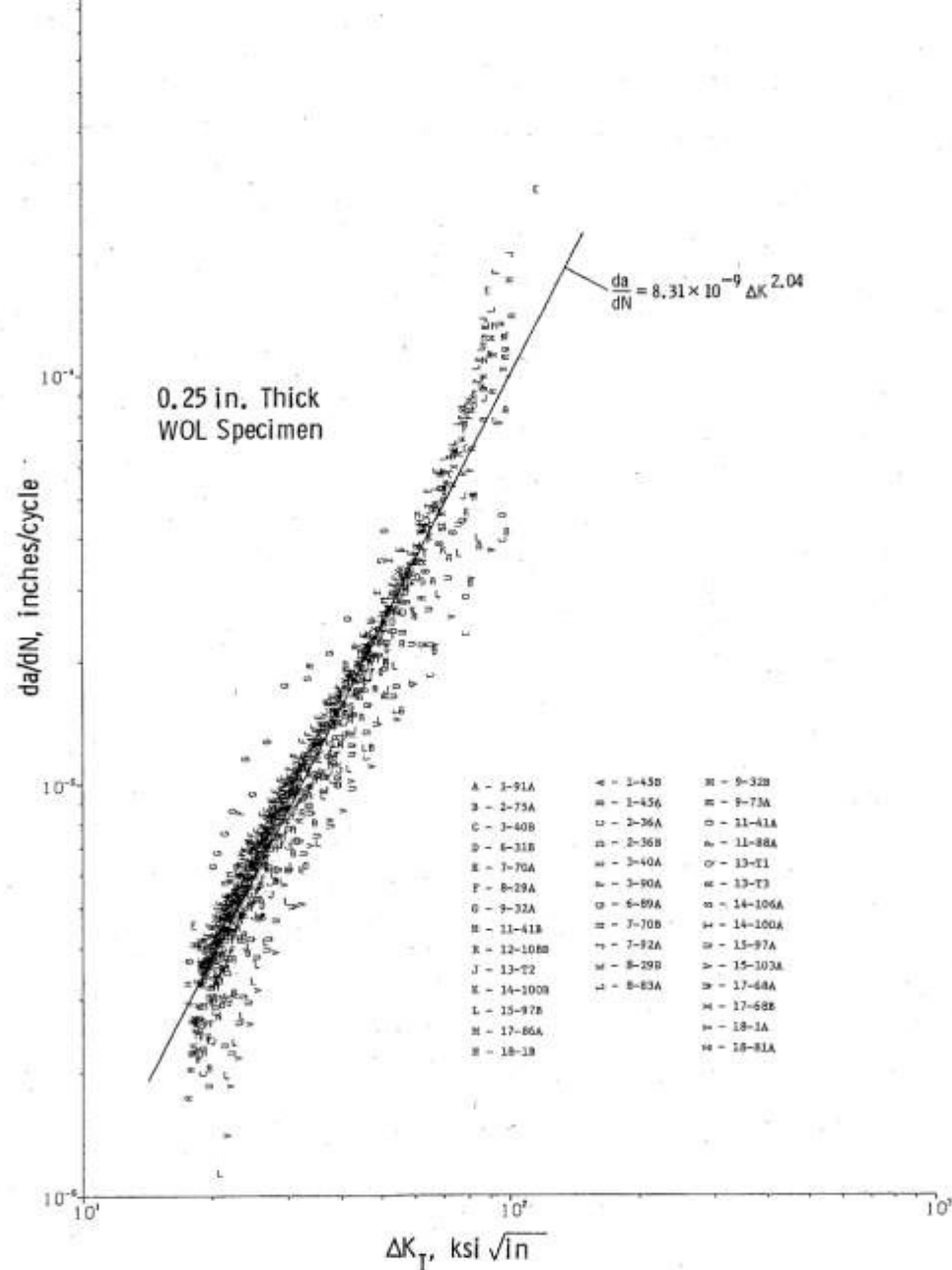


Fig. 7—Summary of Reported Crack Growth Rate Data  
(0.25 in. Thick WOL)

# Structural Steel

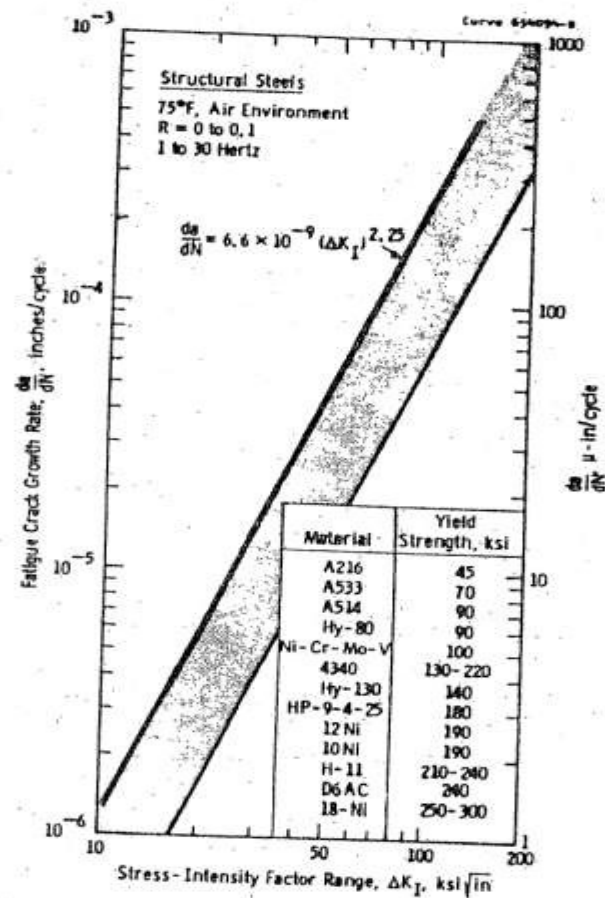


Fig. 26 - Summary of fatigue crack growth rate data for

# A 517 Steel

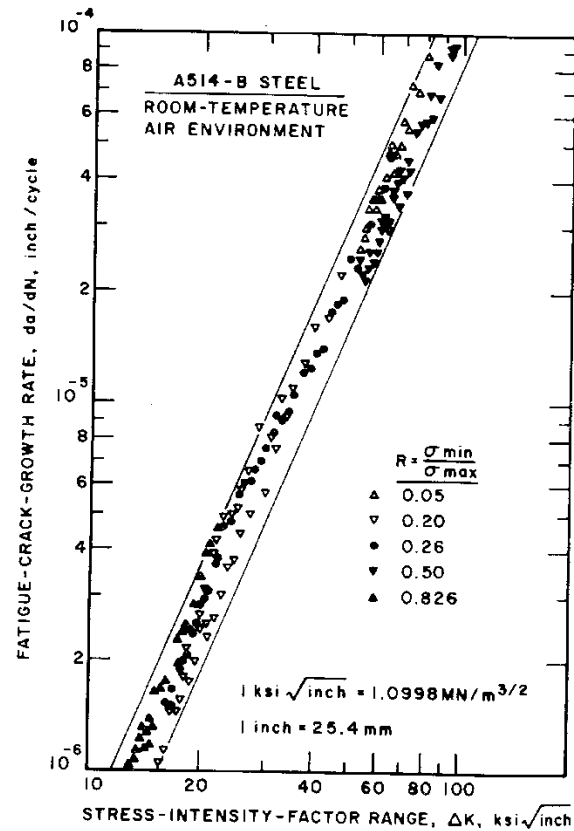


Figure 9.10 Crack-growth rate as a function of stress-intensity-factor range for A514 Grade B steel.



# Aluminum Alloys

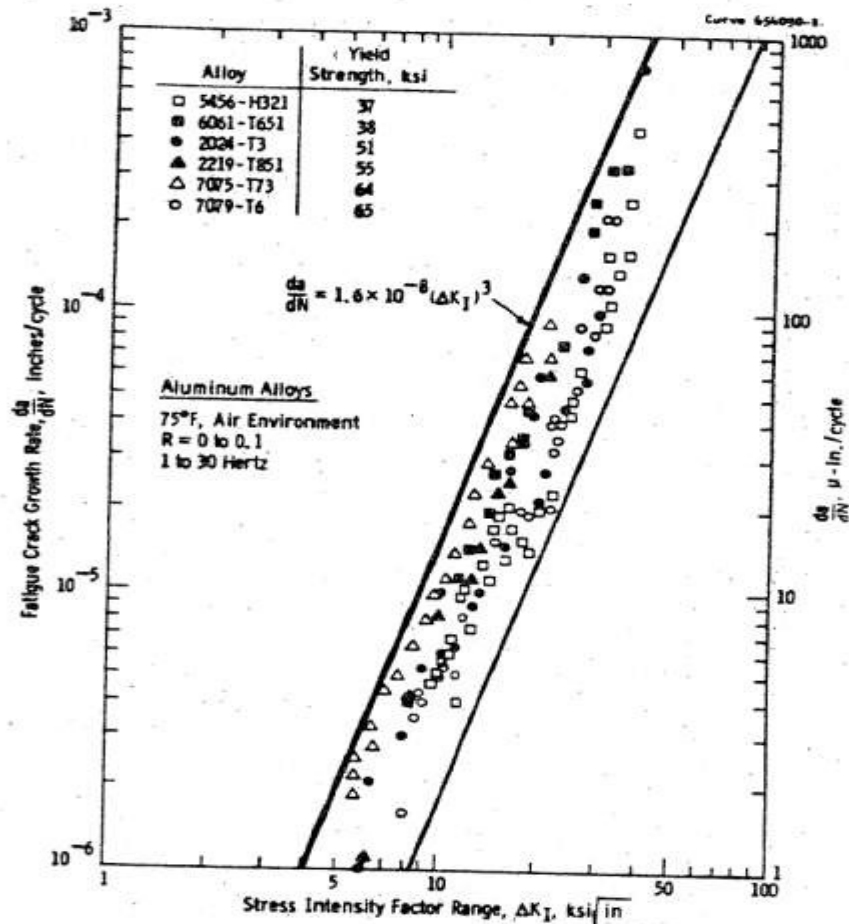


Fig. 27 - Summary data for aluminum alloys.

# Stainless Steel

1000-279

9.5 Austenitic Stainless Steels

291

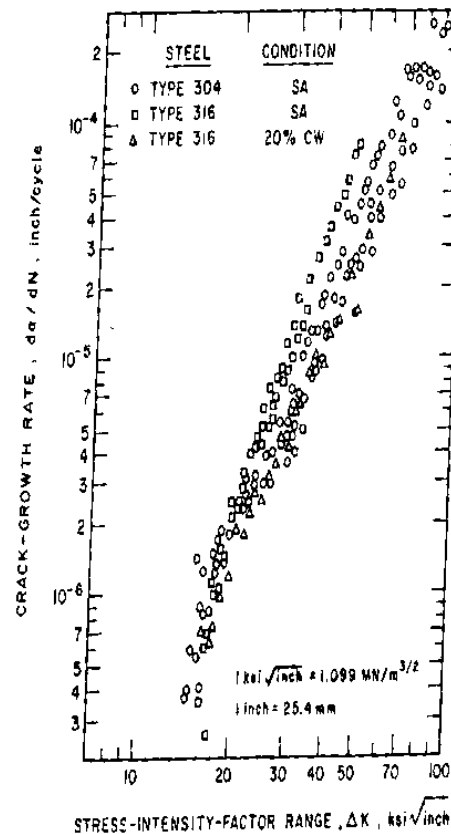


Figure 9.13 Fatigue-crack-growth rate of solution-annealed type 304 and 316 stainless steel and 20 percent cold-worked type 316 stainless steel at 75°F.

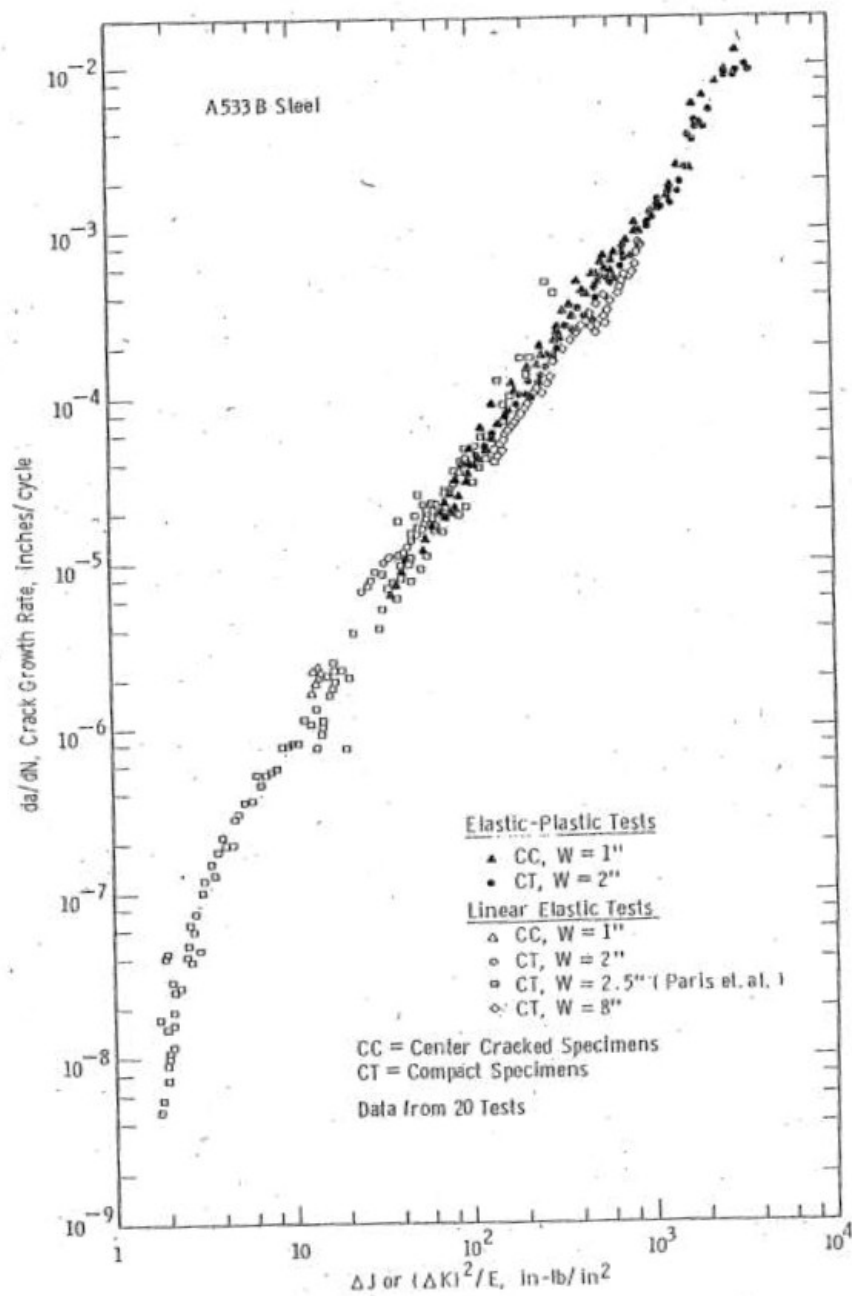


Fig. 9 —Fatigue crack growth rate versus cyclic J for various geometries

# Region 2 $da/dN$ vs $\Delta K$

- Crack growth rates are consistent for a given alloy type, regardless of strength
- Growth rate aligns better with modulus than with strength
- Scatter is likely to be related to test technique

# Sources of Fatigue CGR Data

- Damage Tolerant Handbook
- Barsom and Rolfe

4340

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Condition/Ht: MARTEMPERED

Form: 0.5 in. Plate

Specimen Type: CCP (max stress specified)

Orientation: L-T

Stress Ratio: 0.02

Frequency:

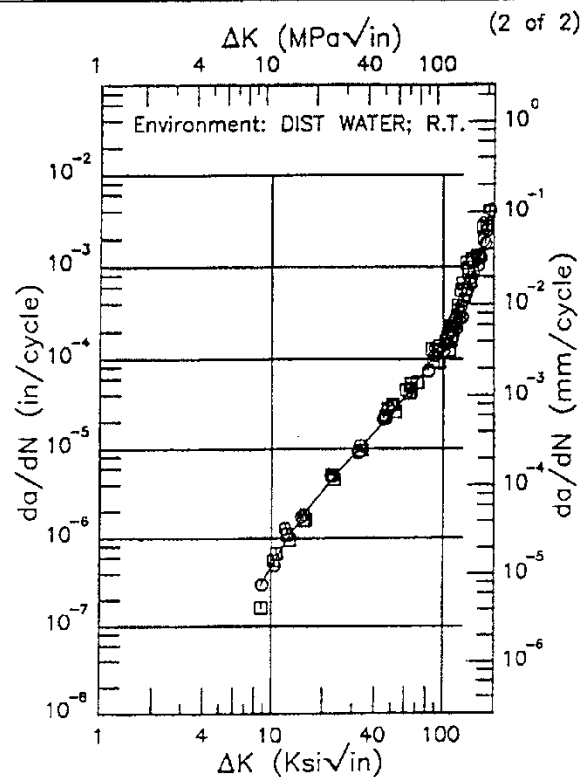
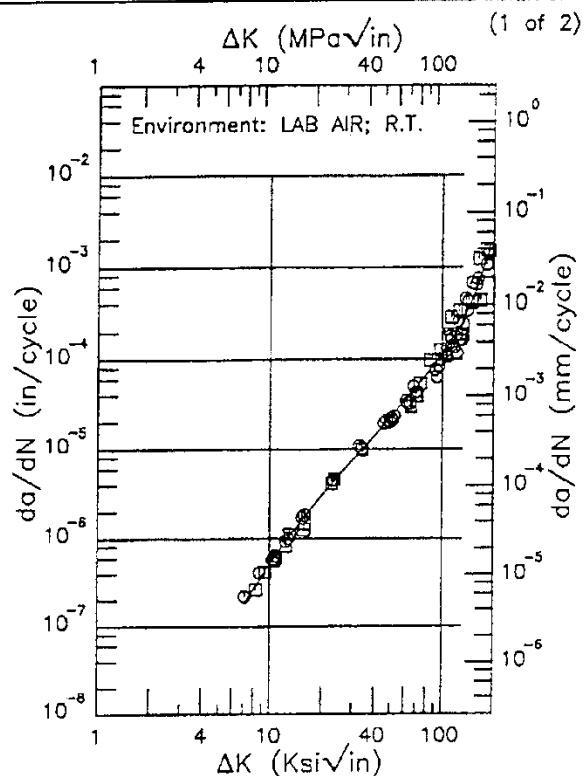
Yield Strength: 191 - 201.5 ksi

Ult. Strength: 196 - 209 ksi

Specimen Thk: 0.246 - 0.251 in.

Specimen Width:

Ref: MA012

 $\Delta K$  (Ksi $\sqrt{\text{in}}$ )da/dN (10<sup>-6</sup> in/cycle)

7.13 (min)	0.183
8.	0.264
9.	0.379
10.	0.516
13.	1.06
16.	1.80
20.	3.06
25.	5.08
30.	7.59
40.	14.1

 $\Delta K$  (Ksi $\sqrt{\text{in}}$ )da/dN (10<sup>-6</sup> in/cycle)

8.73 (min)	0.293
9.	0.329
10.	0.481
13.	1.12
16.	1.98
20.	3.43
25.	5.66
30.	8.34
40.	15.2
50.	24.6

# Effect of R ratio

1. R ratio is

$$R = K_{\min}/K_{\max}$$

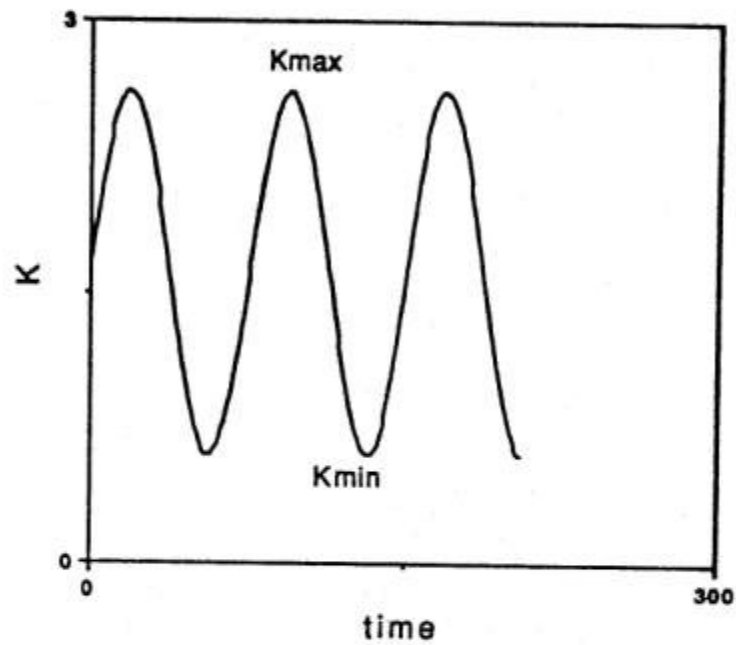
2. The level of R controls the threshold,  
 $\Delta K_{\text{TH}}$

3. The R ratio is important in corrosion fatigue

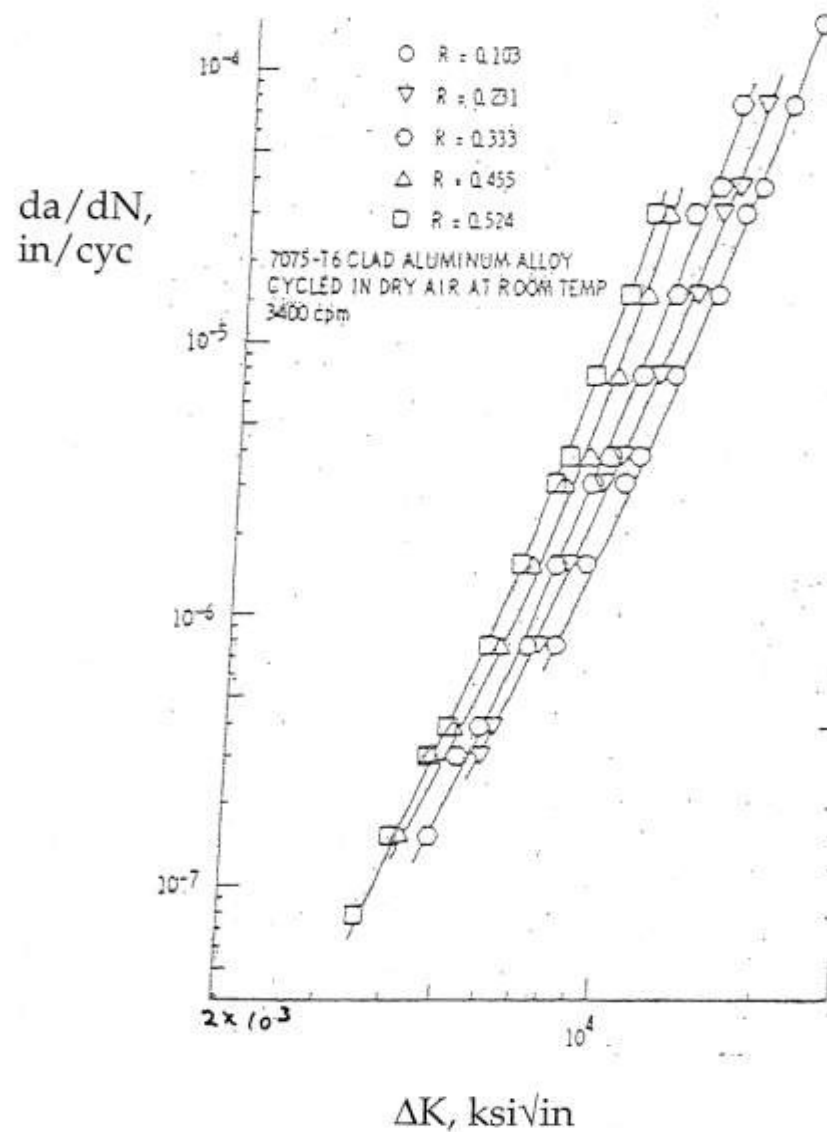
## Effect of R ratio

1. R ratio is

$$R = K_{\min}/K_{\max}$$







Effect of stress ratio,  $R$ , on  $da/dN$   
 7075-T6 Aluminum

# Fatigue crack growth rate equations

1. Paris:

$$\frac{da}{dN} = C\Delta K^n \quad C \text{ and } n \text{ are constants}$$

2. Walker:

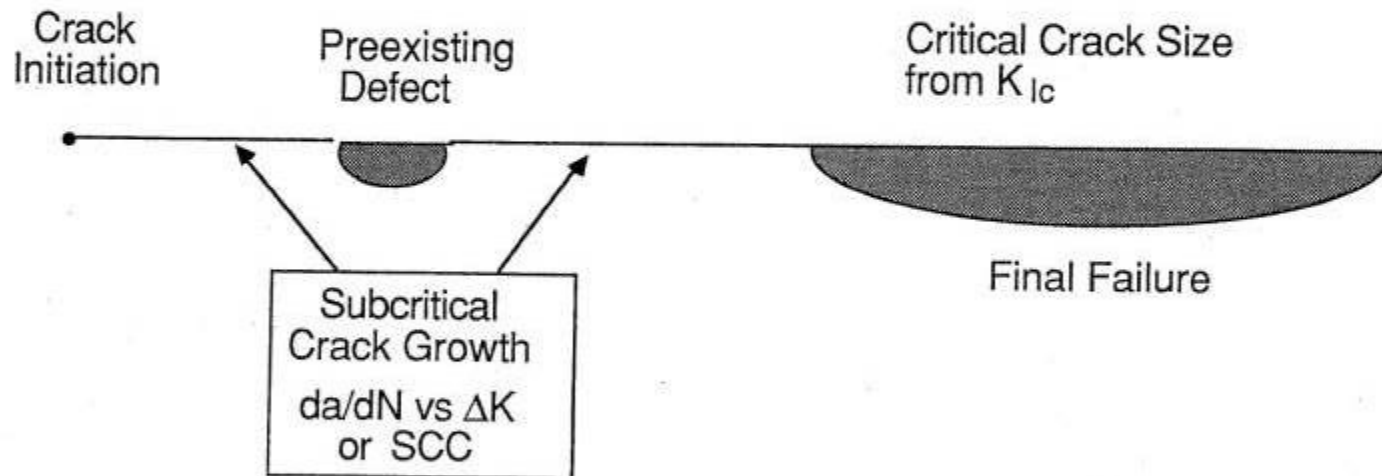
$$\frac{da}{dN} = C_1 \left[ \frac{\Delta K}{(1-R)^{(1-\gamma)}} \right]^{m_1} \quad C_1, n_1, \gamma \text{ are constants}$$

3. Forman:

$$\frac{da}{dN} = \frac{C_2(\Delta K)^{m_2}}{(1-R)K_c - \Delta K} = \frac{C_2(\Delta K)^{m_2}}{(1-R)(K_c - K_{\max})}$$

$C_2, m_2, K_c$  are constants

# Schematic of Life Prediction



## Predicting life from $da/dN$ vs $\Delta K$

1.  $da/dN = C(\Delta K)^n$

2.  $K = \sigma\sqrt{\pi a} F$

3.  $da/dN = C(\Delta\sigma\sqrt{\pi a} F)^n$

4.  $dN = da / C(\Delta\sigma\sqrt{\pi a} F)^n$

5.

$$N = \int_{a_0}^{a_f} \frac{da}{C (\Delta\sigma\sqrt{\pi a} F)^n}$$

6. This is easier if  $F$  is constant or a simple function

# Threshold fatigue

1. In region 1 of  $da/dN$  vs  $\Delta K$ , the crack growth rate slows and growth may stop
2. This is called  $\Delta K_{TH}$ , threshold value of  $\Delta K$  below which there is no crack growth
3.  $\Delta K_{TH}$  may be controlled by crack closure

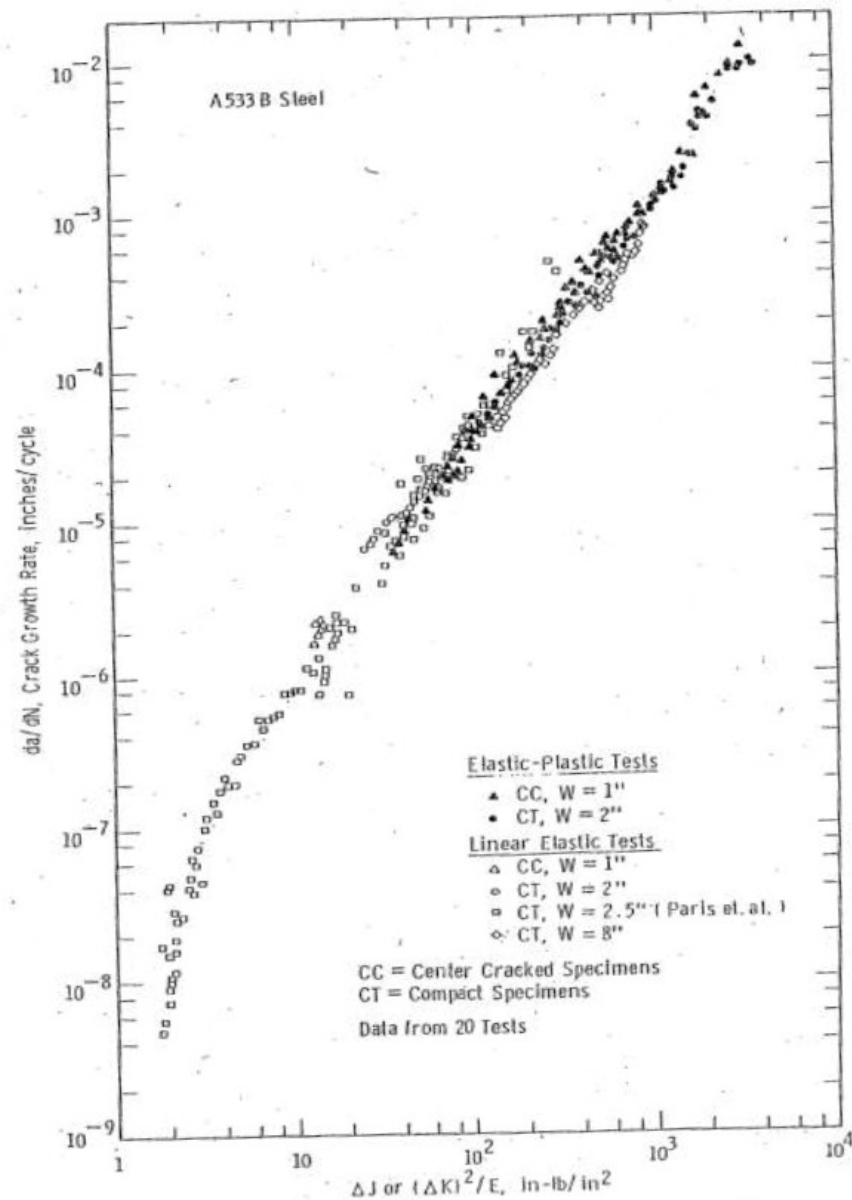


Fig. 9 —Fatigue crack growth rate versus cyclic J for various geometries

# Testing for fatigue threshold

- Standard from ASTM 647
- Crack growth rates:  
 $da/dN < 1 \times 10^{-8} \text{ m/cyc}$  ( $4 \times 10^{-7} \text{ in/cyc}$ )
- Test usually conducted with a decreasing  $\Delta K$  value

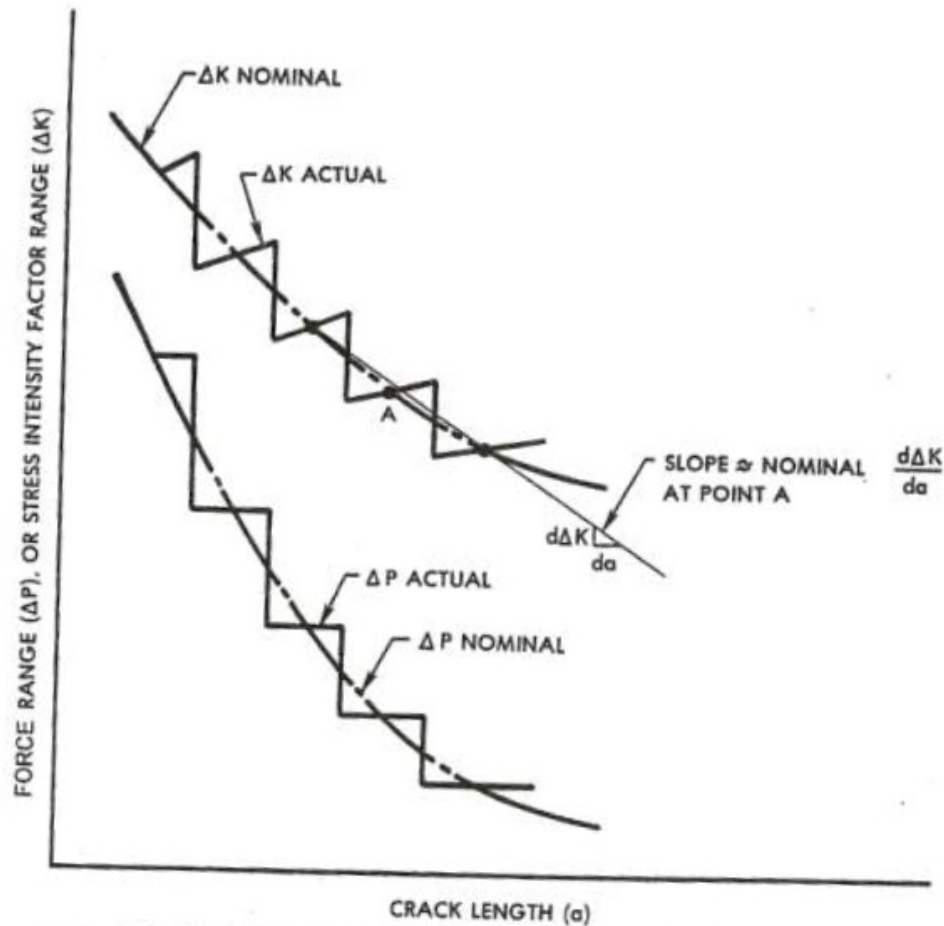


FIG. 2 Typical  $K$  Decreasing Test by Stepped Force Shedding



# Decreasing $\Delta K$ test -N

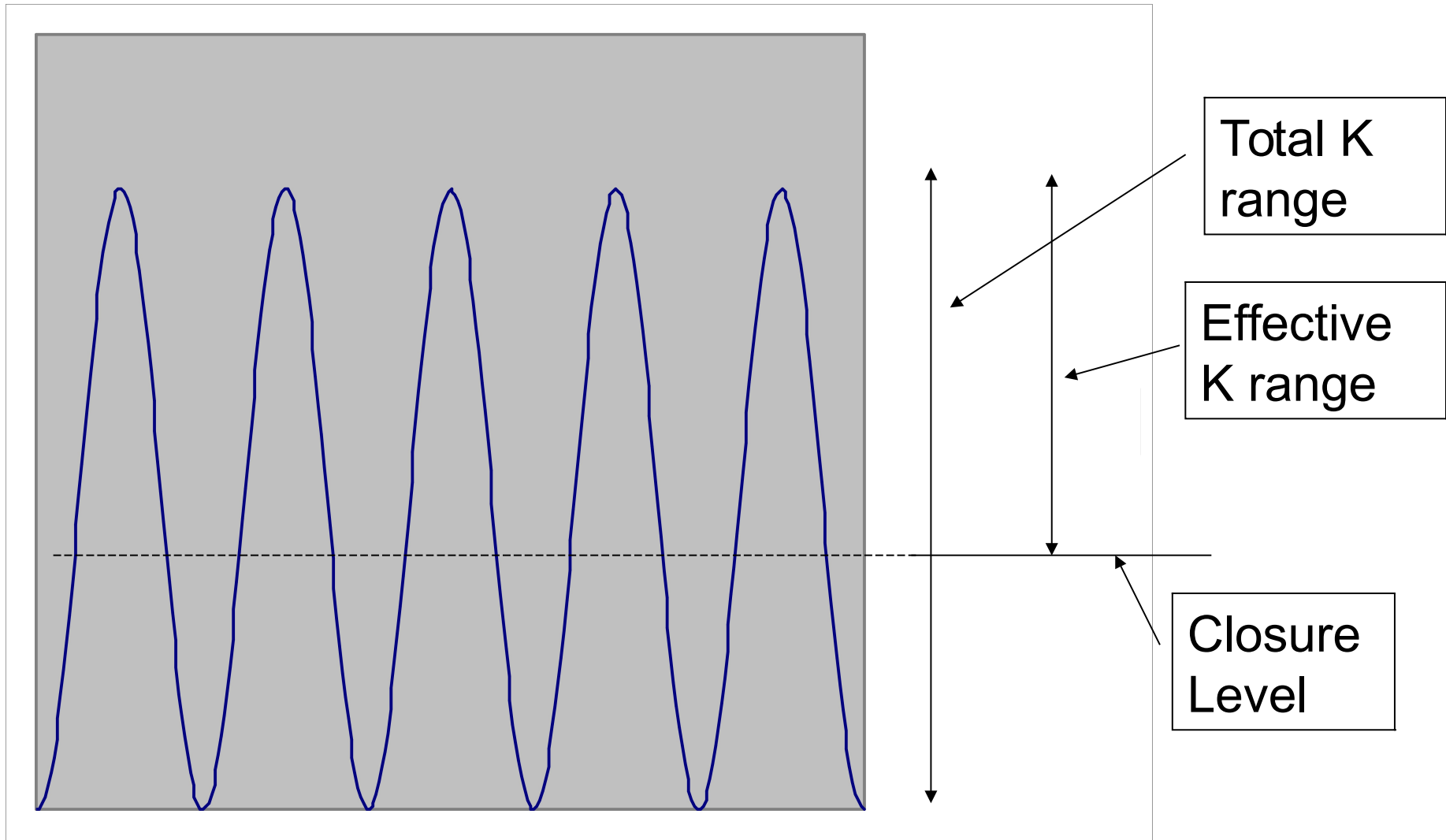
- Manually;
  - Decrease the load,  $\Delta P$  after a small amount of crack growth
  - Continue until no crack growth after X no. of cycles
- Computer controlled
  - Program a continuous decrease in  $\Delta K$
  - Program this function with  $C \sim -0.1/\text{mm}$

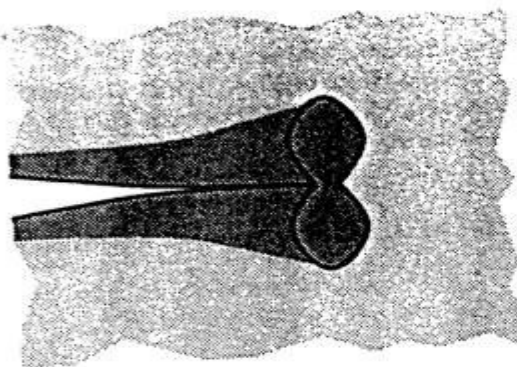
$$\Delta K = \Delta K_o \exp[C(a - a_o)]$$

# Fatigue threshold related to crack closure

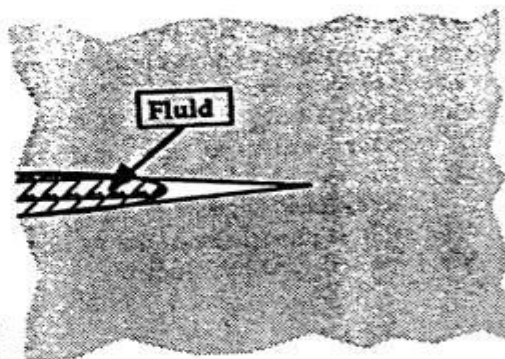
- During the fatigue cycle the crack may close before zero load
- Early closure decreases the effective  $K$  of the fatigue cycle

# Effect of Crack Closure on K Range

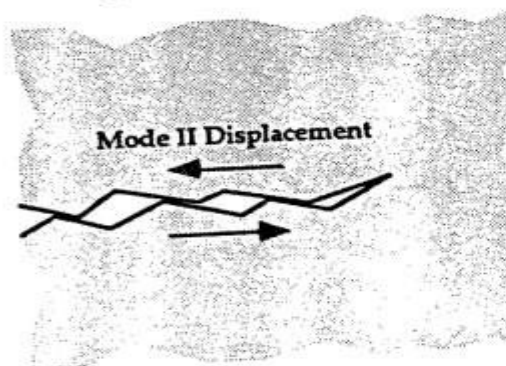




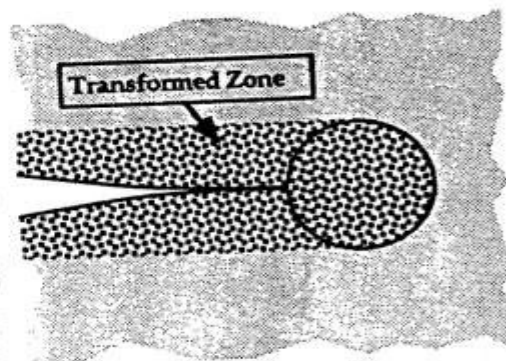
(a) Plasticity-induced closure.



(d) Closure induced by a viscous fluid.



(b) Roughness-induced closure.



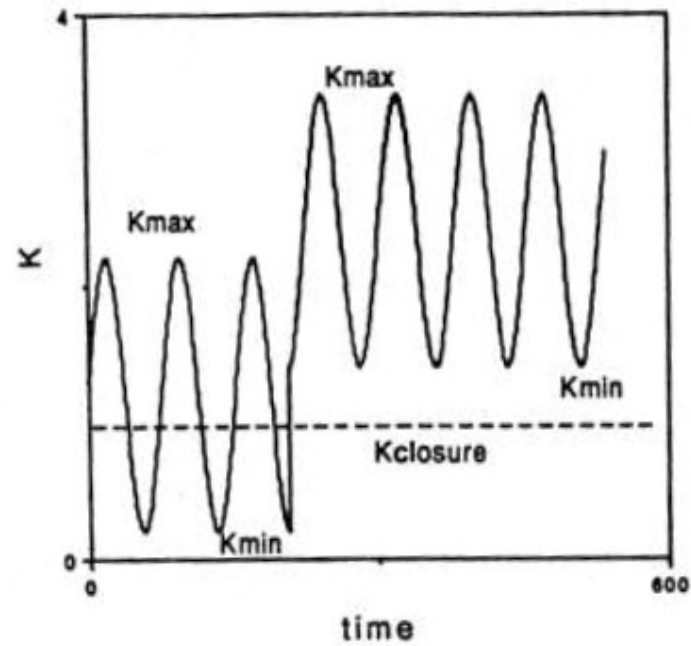
(e) Transformation-induced closure.



(c) Oxide-induced closure.

FIGURE 10.5 Fatigue crack closure mechanisms in metals [14].

2. The level of R controls the threshold and hence the  $\Delta K_{TH}$



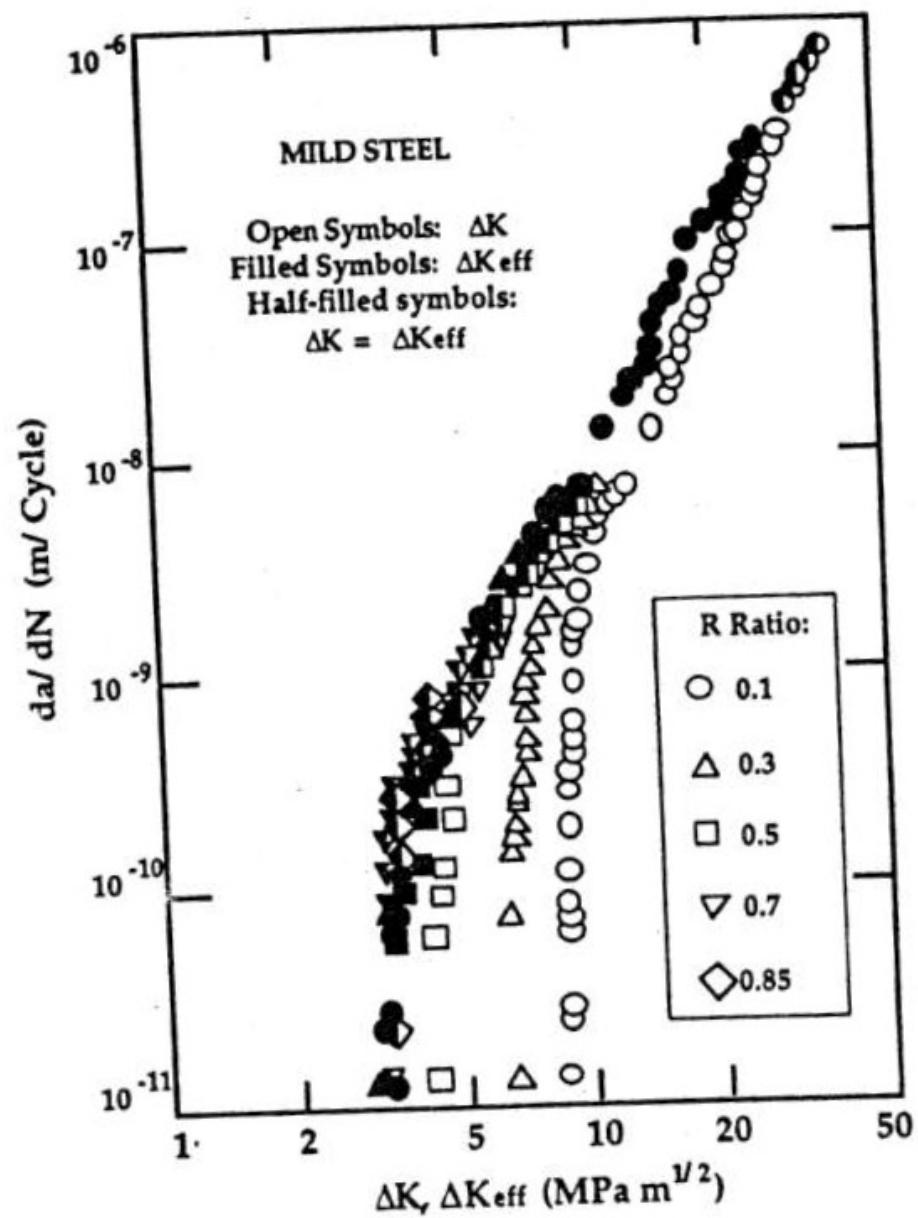


FIGURE 10.8. Fatigue crack growth data for mild steel at various  $R$  ratios [7].

### Fatigue Crack Propagation

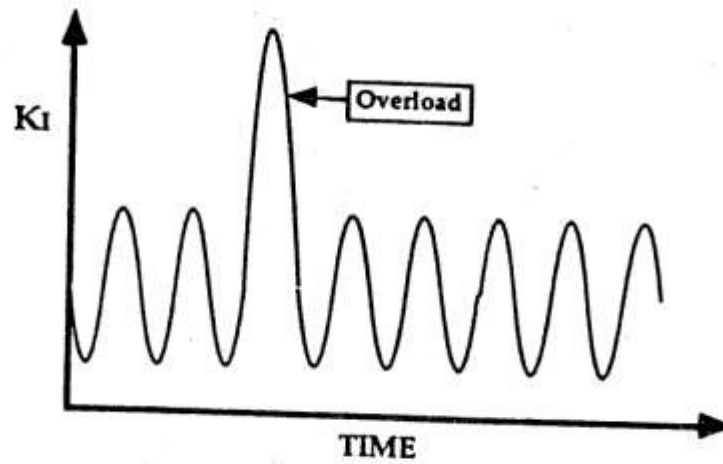


FIGURE 10.12 A single overload during cyclic loading.

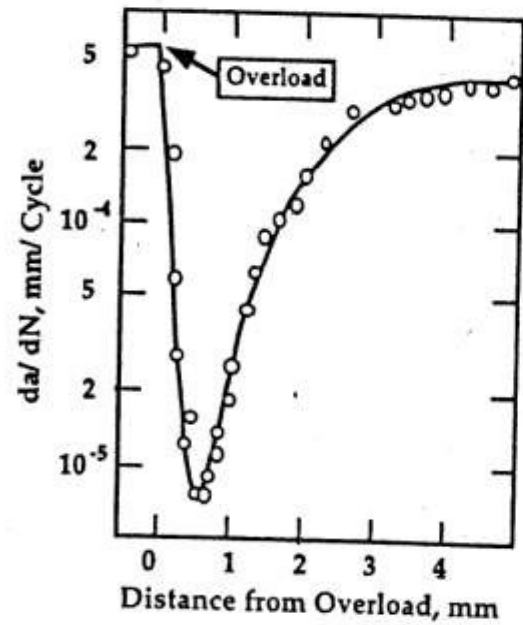
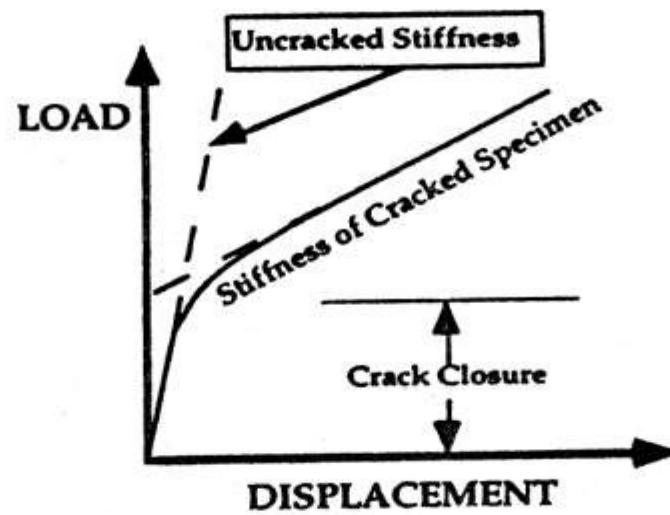
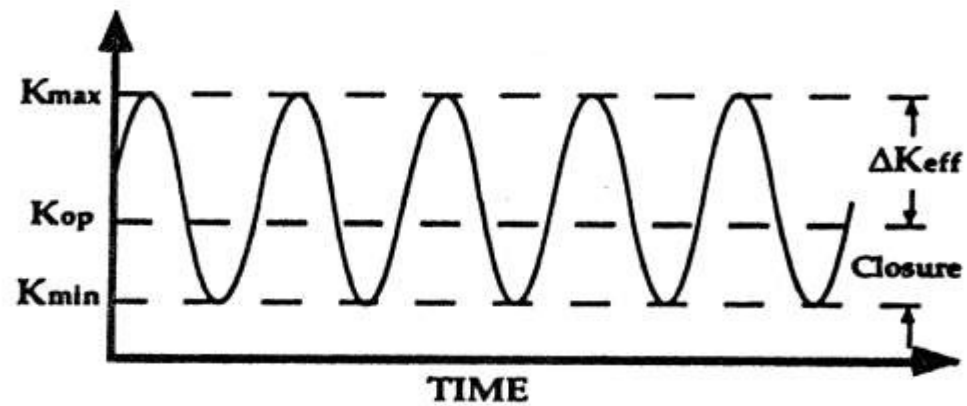


FIGURE 10.13 Retardation of crack growth following an overload [33].

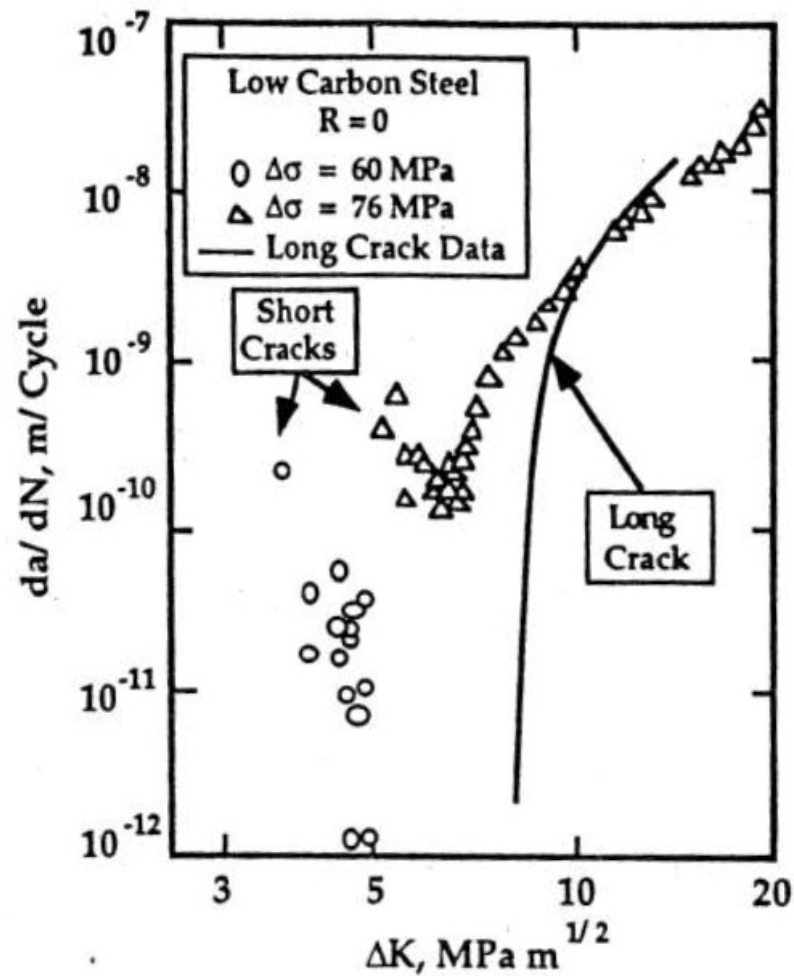


Load vs displacement with crack closure



Effect of crack closure





Effect of short cracks on  $da/dN$  vs  $\Delta K$  results