

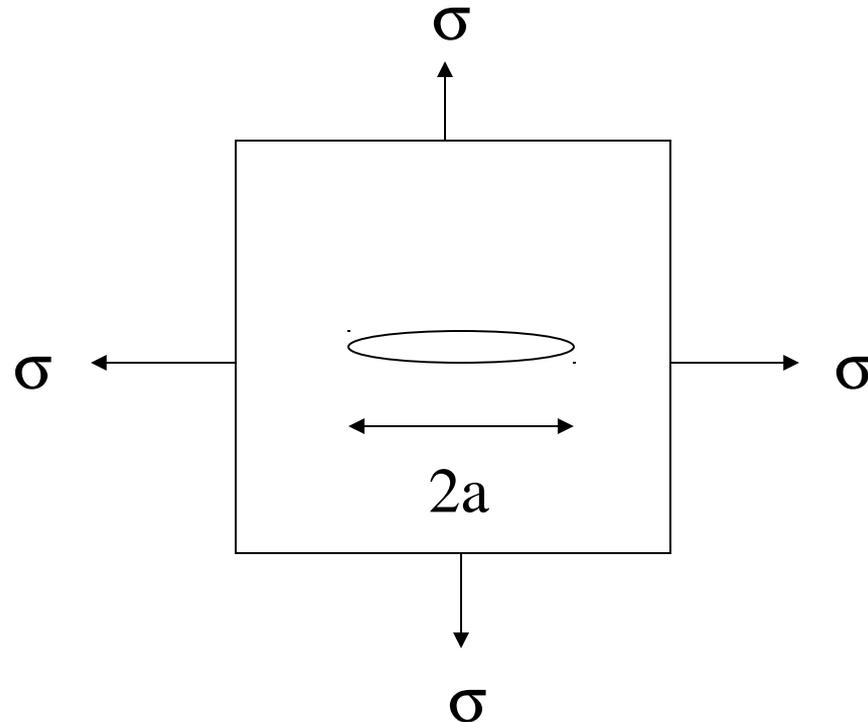
# Lecture 2.1

Analytical tools

# Griffith Fracture Theory

First acknowledged fracture theory

Energy balance on stressed plate with crack



# Griffith Fracture Theory, Cont'd

## Potential energy calculation

$$U = U_o + U_a + U_t = \text{Total Potential Energy}$$

where

$U_o$  = P.E. of plate with no crack

$U_a$  = P.E. due to crack =  $-\pi\sigma^2a^2t/E$

$U_t$  = surface energy per unit area =  $4atT$

$$U = 4atT - \pi\sigma^2a^2t/E + U_o$$

For equilibrium:

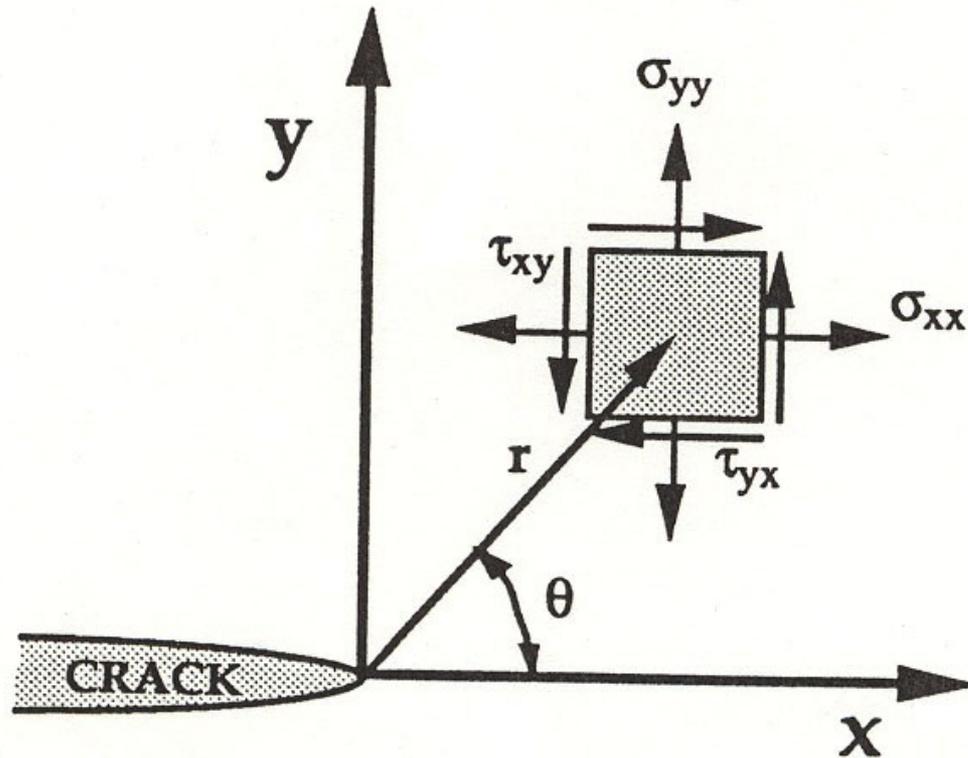
$$dU/da = 0 = 4tT - 2\pi\sigma^2at/E$$

$$\pi\sigma^2a/E > 2T \quad \text{Unstable}$$

# Griffith Fracture Theory, Cont'd - N

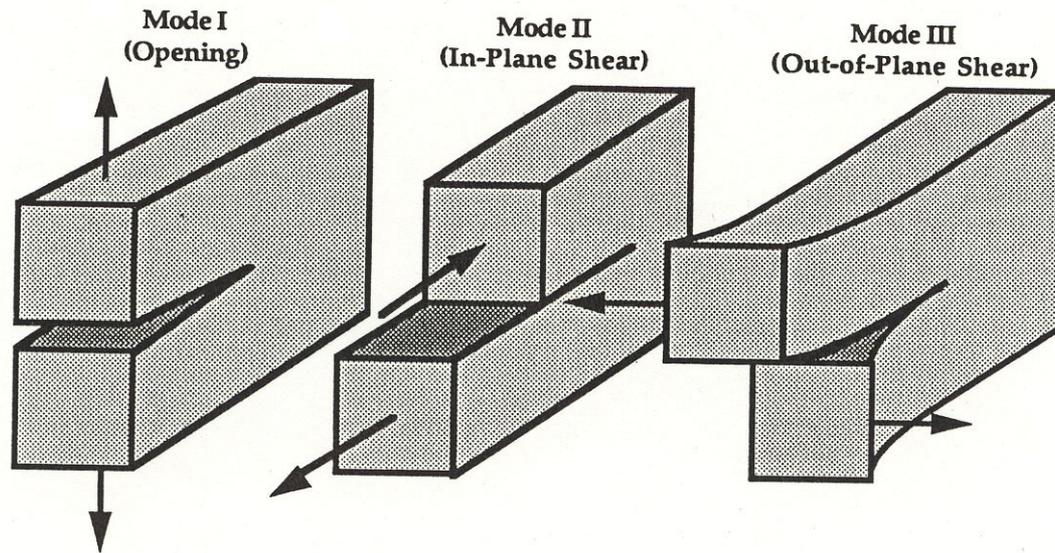
- A Griffith's energy term was labeled  $G$
- $G = 2T = \pi\sigma^2a/E$
- This term was later related to Irwin's stress intensity factor,  $K$
- The Griffith approach was developed in the 1920's
- A new approach was proposed until the 1950's this came in the form of the Irwin  $K$

# The Stress Intensity Factor



# The Stress Intensity Factor, Cont'd

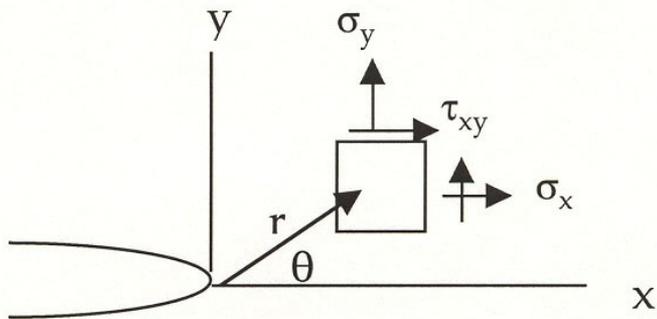
A crack can be loaded in one of three ways:



The stress fields ahead of a crack tip in a linear elastic material are proportional to  $1/\sqrt{r}$ . The proportionality constant is called the *stress intensity factor*.

# Fracture Mechanics

## 1. Linear Elastic



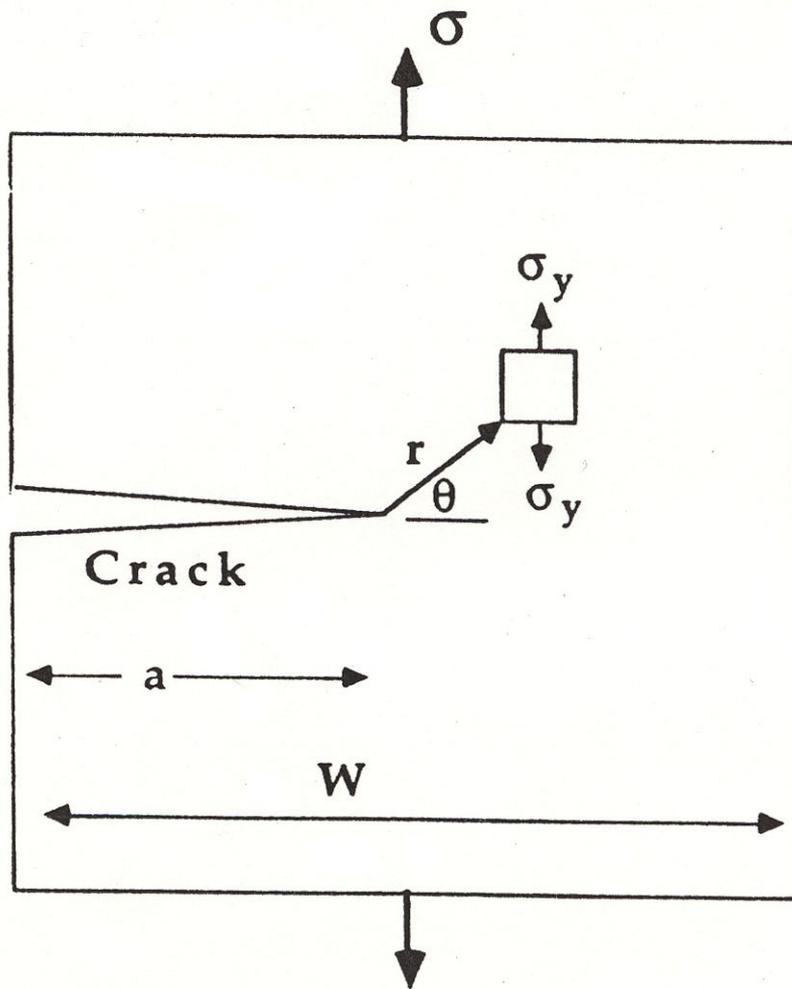
$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2}$$

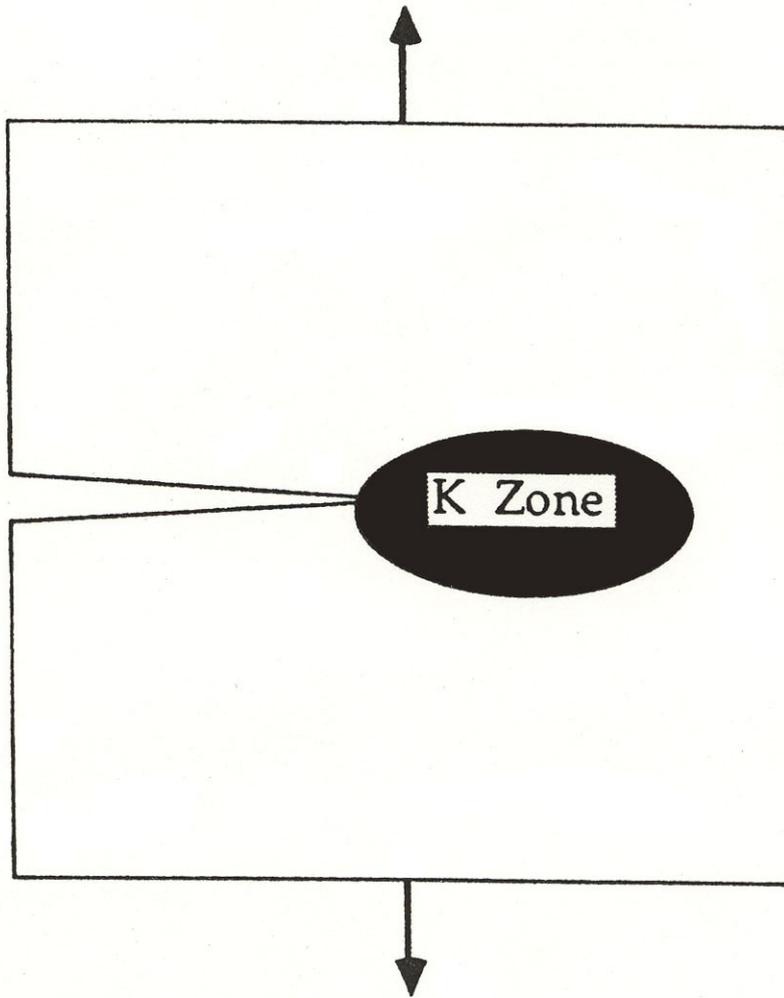
# Crack-tip stress analysis - N

- Two main features
  - Magnitude,  $K_I$ , indicator of a happening, fracture, fatigue, etc.
  - Distribution – everything else,  
unique for mode I
- $K$  has a key role



$$\sigma_y = \frac{K}{\sqrt{2\pi r}} f(\theta)$$

$$K = \sigma \sqrt{\pi a} f(a/W)$$



1. Crack-tip stress field has unique distribution
2. Magnitude of field is given by fracture parameter  $K$

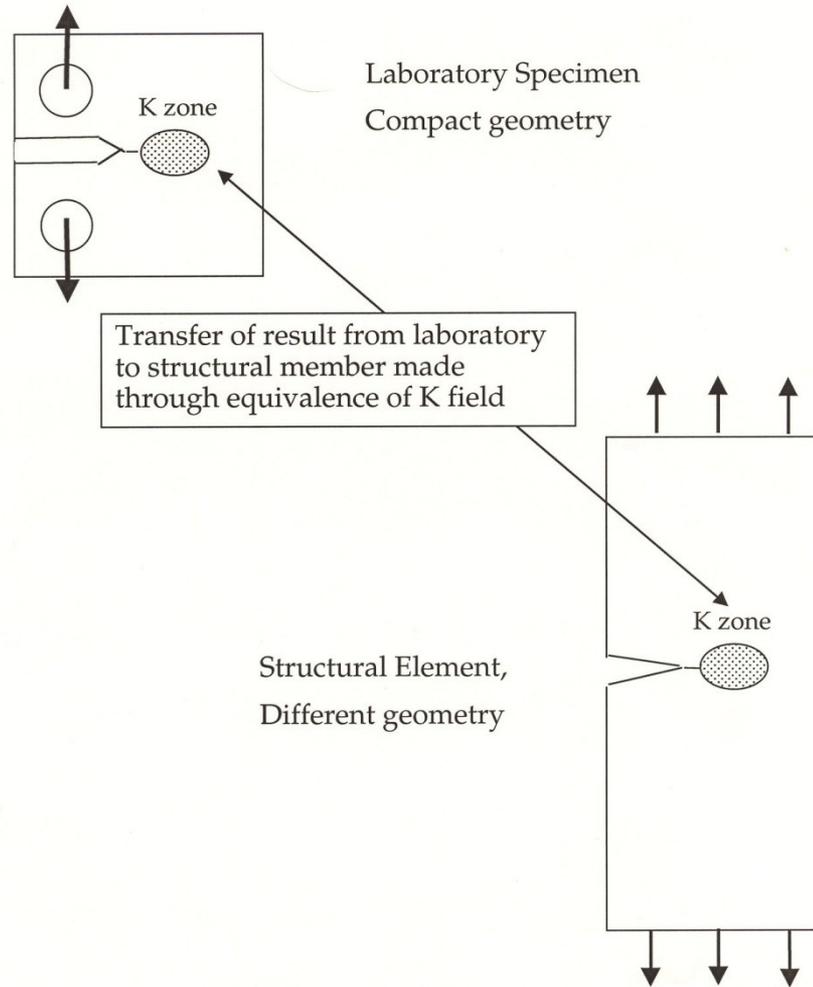
# Summary on Fracture Criteria

1. Griffith energy balance,  $G \rightarrow G_{cr}$ , is equivalent to crack-tip field criterion,  $K \rightarrow K_{cr}$
2.  $K$  comes from a stress analysis and may be easier to find
3. The expression

$$G = \frac{K^2 (1 - \nu^2)}{E} = \frac{P^2}{2B} \frac{dC}{da}$$

links  $K$  to energy release rate

# Basis of Fracture Mechanics



# Review of Crack-Tip Field Approach

1. The crack-tip field has a unique stress-strain distribution
2. The magnitude of the field depends on  $K$
3. The boundary loading relates to the crack-tip stress field through  $K$
4. The condition for failure is phenomenological “black box” approach: the entire field reaches a critical value; no mechanism needs to be cited
5. To predict failure or fracture events, there is a need to determine  $K$

# Types of Fracture Behavior

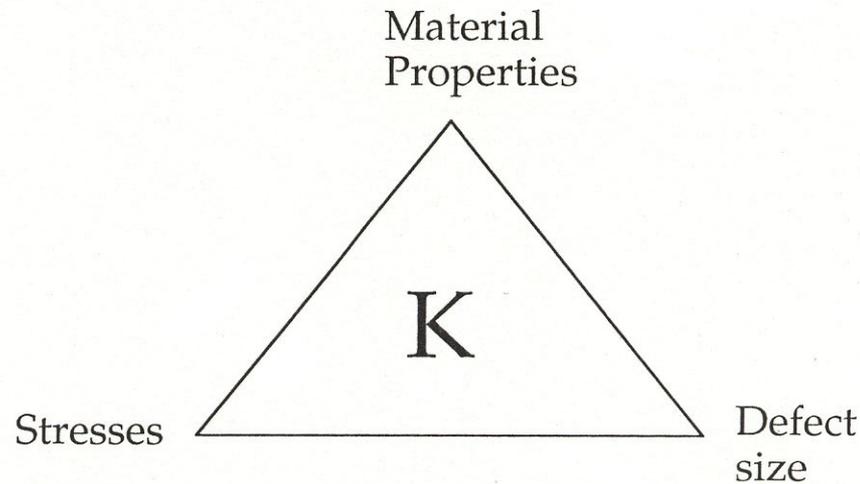
1. Fracture toughness
2. Fatigue crack growth
3. Stress corrosion cracking
4. Creep cracking

# Applications

1. Reasons for applications
  - Design criteria
  - Material selection
  - Failure analysis

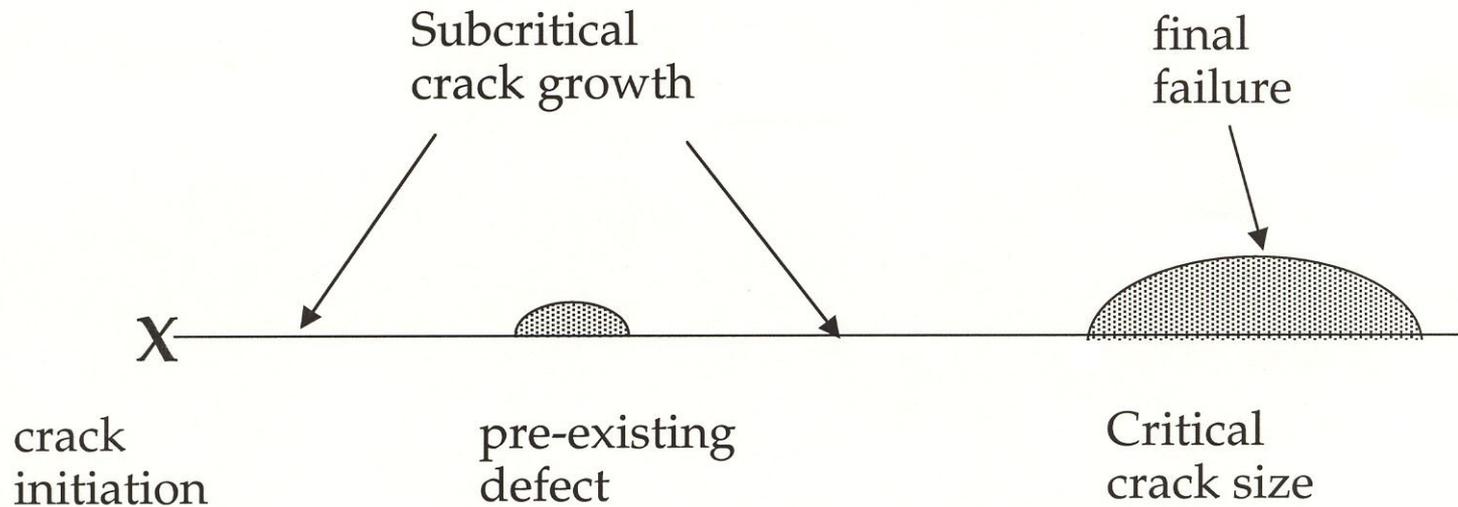
# Applications, Cont'd

## 2. Use the FM triangle as a guide



Given two corners, predict the third

# Schematic of Life Prediction



Life of a structure containing a crack-like defect

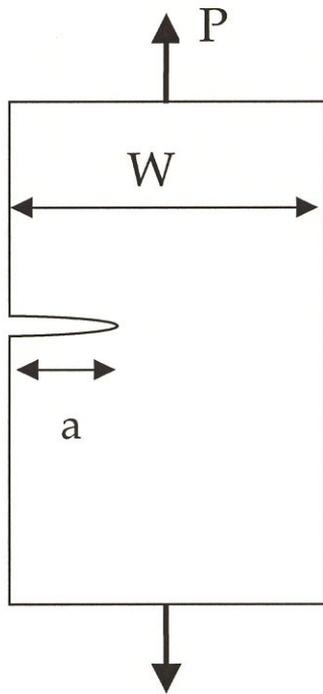
# Methods for Finding K

1. Closed form analysis - Theory of elasticity solutions
2. Numerical - Finite element and others
3. Experimental - Photoelasticity, strain gages, etc.
4. Estimation
5. Handbook

# K solution forms

## 2. K solutions

Handbooks, standards



$$K = \sigma \sqrt{\pi a} F\left(\frac{a}{W}\right)$$

$$K = \frac{Pf(a/W)}{B\sqrt{W}}$$

# K solution - N

- Units
  - Eng: ksi-in<sup>1/2</sup>
  - SI: MPa-m<sup>1/2</sup>

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THE CENTER CRACKED TEST SPECIMEN

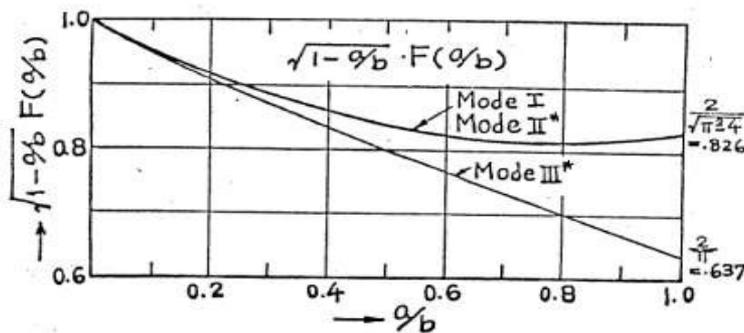
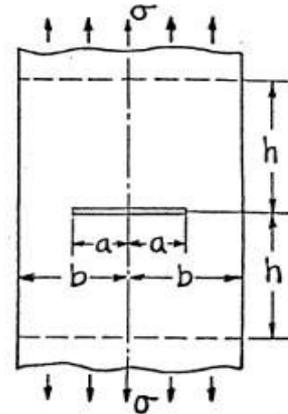
A. Stress Intensity Factor

$$K_I = \sigma \sqrt{\pi a} F(a/b)$$

Numerical Values at F(a/b)

(Isida 1962, 1965 a, b, 1973)

Isida's 36 term power series of  $(a/b)^2$  (Laurent series expansion of complex stress potential, 1973) gives practically exact values of  $F(a/b)$  up to  $a/b = 0.9$ . Numerical values of  $F(a/b)$  are shown in the table.



<u>a/b</u>	<u>F(a/b)</u>
0.0	1.0000
0.1	1.0060
0.2	1.0246
0.3	1.0577
0.4	1.1094
0.5	1.1867
0.6	1.3033
0.7	1.4882
0.8	1.8160
0.9	2.5776
1.0	$\frac{2}{\sqrt{\pi \cdot 2.4} \cdot \sqrt{1-a/b}}$ **

Empirical Formulas

- a. Accuracy  
b. Method of derivation, reference

$$F(a/b) = \sqrt{\frac{2b}{\pi a} \tan \frac{\pi a}{2b}}$$

- a. Better than 5% for  $a/b \leq 0.5$   
b. Approximation by periodic crack solution,  
Irwin 1957

$$F(a/b) = 1 + 0.128(a/b) - 0.288(a/b)^2 + 1.525(a/b)^3$$

- a. 0.5% for  $a/b \leq 0.7$   
b. Least square fitting to Isida's results,  
Brown 1966

$$F(a/b) = \sqrt{\sec \frac{\pi a}{2b}}$$

- a. 0.3% for  $a/b \leq 0.7$ , 1% at  $a/b = 0.8$   
b. Guess based on Isida's results, Feddersen-1966

$$F(a/b) = \frac{1 - 0.5(a/b) + 0.326(a/b)^2}{\sqrt{1 - a/b}}$$

- a. 1% for any  $a/b$   
b. Asymptotic Approximation, Koiter 1965 b

$$F(a/b) = \frac{1 - 0.5(a/b) + 0.370(a/b)^2 - 0.044(a/b)^3}{\sqrt{1 - a/b}}$$

- a. 0.3% for any  $a/b$   
b. Modification of Koiter's formula, Tada 1973

$$F(a/b) = \left\{ 1 - 0.025(a/b)^2 + 0.06(a/b)^4 \right\} \sqrt{\sec \frac{\pi a}{2b}}$$

- a. 0.1% for any  $a/b$   
b. Modification of Feddersen's formula, Tada 1973

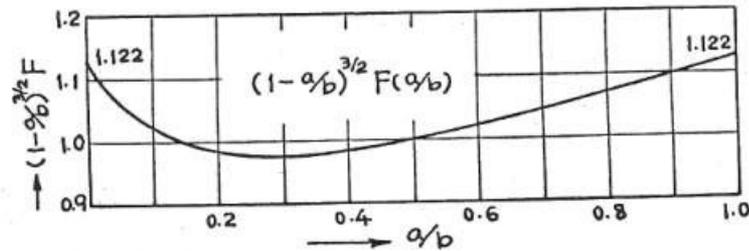
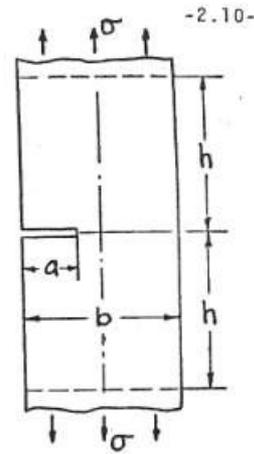
## THE SINGLE EDGE NOTCH TEST SPECIMEN

### A. Stress Intensity Factor

$$K_I = \sigma \sqrt{\pi a} F(a/b)$$

### Numerical Values of $F(a/b)$

The curve in the following figure was drawn based on the results having better than 0.5% accuracy.

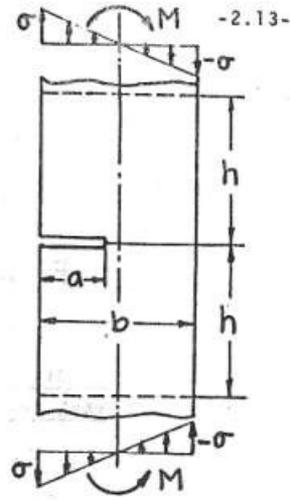


### Methods and References

1. Boundary Collocation Method ( $h/b > 0.8$ ): Gross 1964
2. Mapping Function Method ( $h/b = 1.53$ ): Bowie 1965
3. Green's Function Method ( $h/b > 1.5$ ): Emery 1969, 1971
4. Weight Function Method: Bueckner 1970, 1971
5. Asymptotic Approximation: Benthem 1972
6. Finite Element Method ( $h/b=2.75, 1.0$ ): Yamamoto 1972

### Note

1. Load is applied along the center line of the strip at the crack location (or Uniform Pressure on crack surface).
2. The effect of  $h/b$  is practically negligible for  $h/b \geq 1.0$ .



A. Stress Intensity Factor

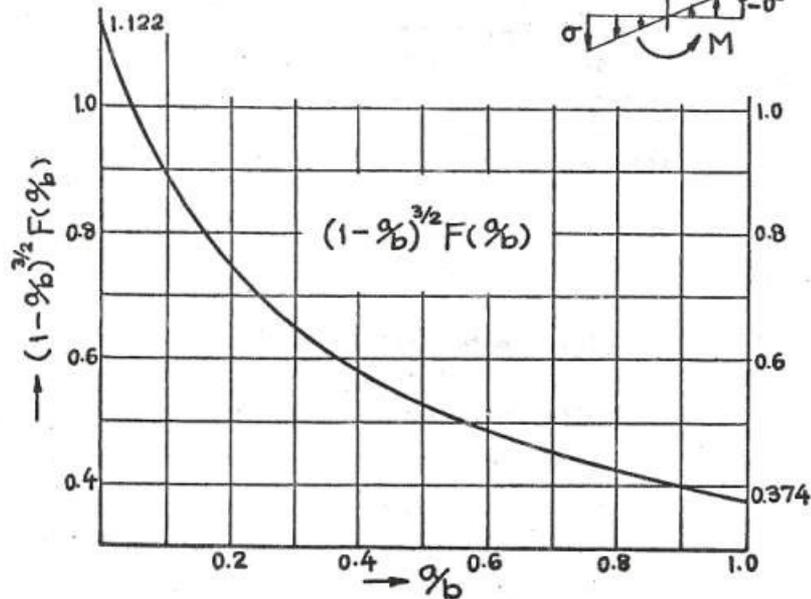
$$\sigma = \frac{6M}{b^2}$$

$$K_I = \sigma \sqrt{\pi a} F(a/b)$$

Numerical Values of F(a/b)

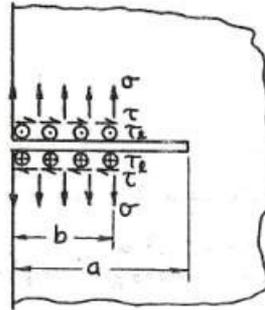
The curve in the following figure was drawn based on the results having better than 0.5% accuracy.

Used also: for 4 point bending.



Methods and References

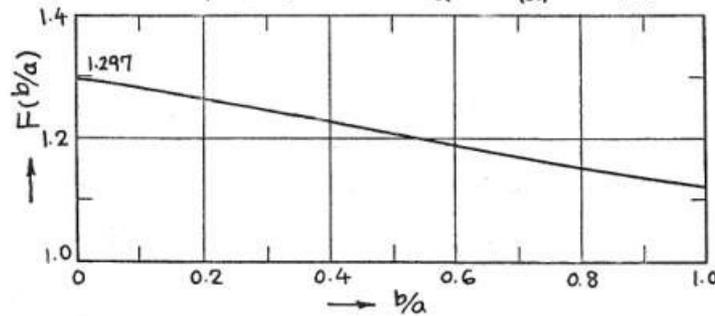
1. Singular Integral Equation, Bueckner 1960
2. Boundary Collocation Method ( $h/b \geq 2$ ), Gross 1965
3. Weight Function Method, Bueckner 1970, 1971
4. Green's Function Method ( $h/b \geq 1.5$ ), Emery 1969
5. Asymptotic Approximation, Benthem 1972



$$\begin{Bmatrix} K_I \\ K_{II} \\ K_{III} \end{Bmatrix} = \begin{Bmatrix} \sigma \\ \tau \\ \tau_x \end{Bmatrix} \sqrt{\pi a} \cdot \frac{2}{\pi} \sin^{-1} \frac{b}{a} \begin{Bmatrix} F(b/a) \\ F(b/a) \\ 1 \end{Bmatrix}$$

$$F(b/a) = 1.3 - 0.18 \frac{b}{a} \quad (1)$$

$$\text{or } F(b/a) = 1.3 - 0.143 \frac{b}{a} - 0.120 \left(\frac{b}{a}\right)^2 + 0.083 \left(\frac{b}{a}\right)^3 \quad (2)$$



Methods:  $K_I, K_{II}$  Alternating Method or Integration of page 8.3

$K_{III}$  Westergaard Stress Function, etc.

Accuracy: Empirical Formula (1) 0.5%, (2) 0.2%

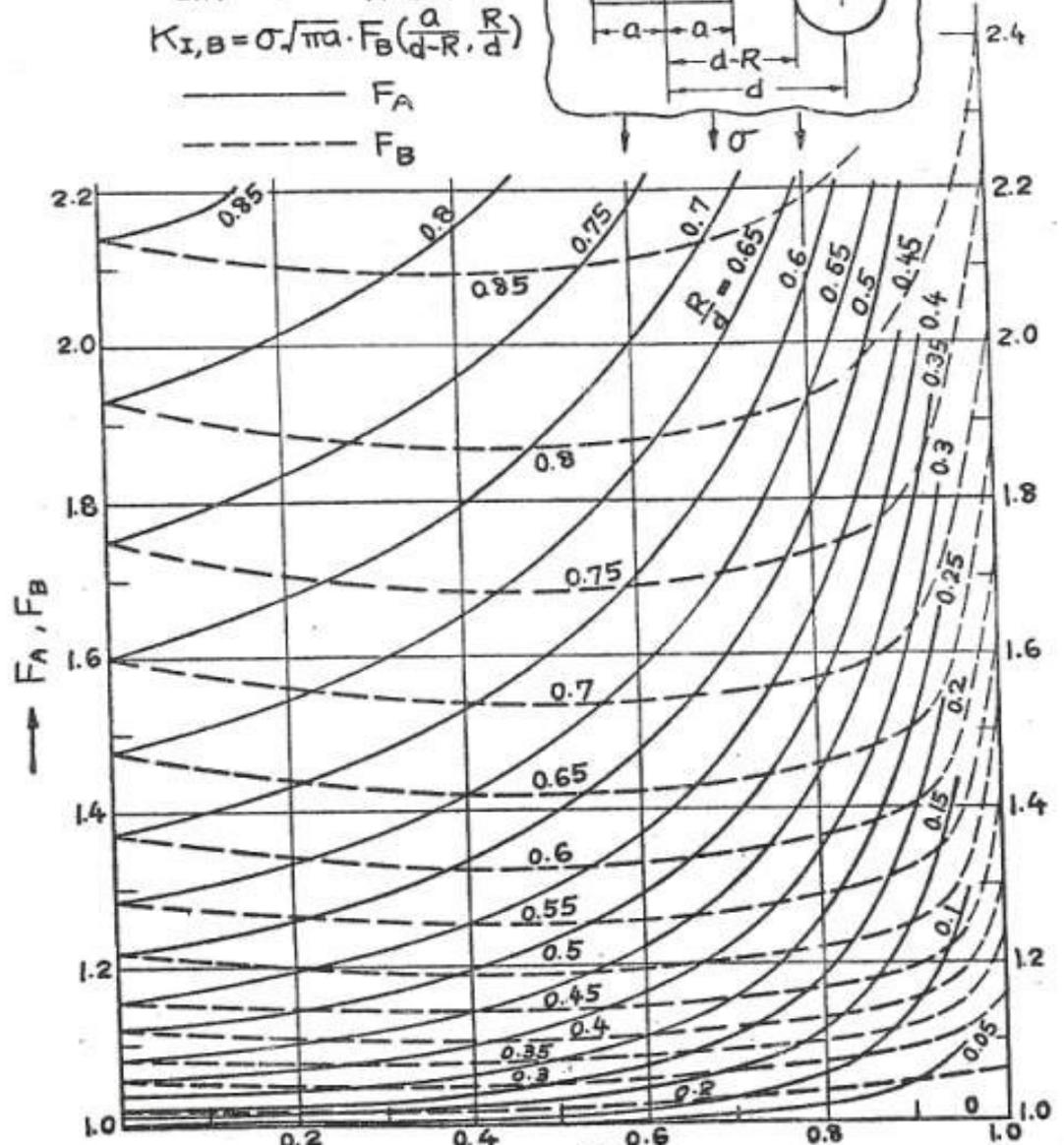
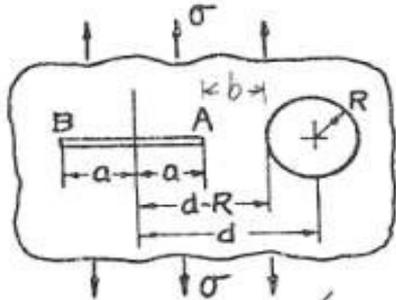
$K_{III}$  Exact

References: Sih 1973, Tada 1985

$$K_{I,A} = \sigma \sqrt{\pi a} \cdot F_A \left( \frac{a}{d-R}, \frac{R}{d} \right)$$

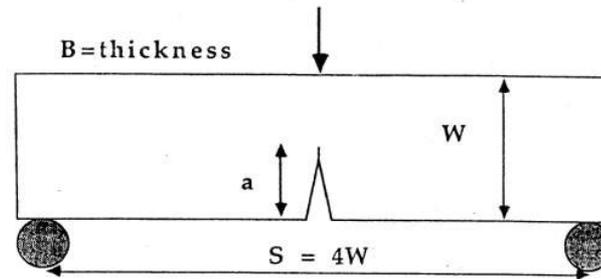
$$K_{I,B} = \sigma \sqrt{\pi a} \cdot F_B \left( \frac{a}{d-R}, \frac{R}{d} \right)$$

—  $F_A$   
 - - -  $F_B$

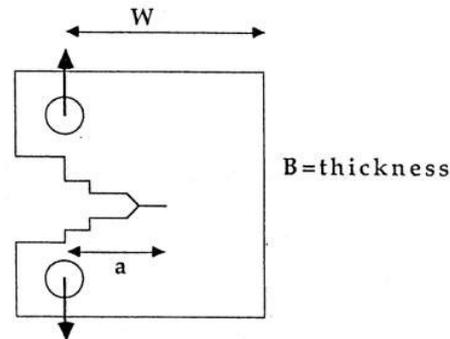


# Fracture Toughness Test Specimens

Fracture toughness test specimens



Single Edge Notched Bend Specimen  
with Three Point Loading



Compact Specimen  
with Pin and Clevis Loading

# Compact Specimen (CT) K solution

$$K = \frac{Pf\left(\frac{a}{W}\right)}{BW^{1/2}}$$

$$f\left(\frac{a}{W}\right) = \frac{\left(2 + \frac{a}{W}\right) \left[ 0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4 \right]}{\left(1 - \frac{a}{W}\right)^{\frac{3}{2}}}$$

	a/W	KB $\sqrt{W/P}$	BEv/P	a/W	KB $\sqrt{W/P}$	BEv/P
1	0.202	4.299	8.702	0.302	5.650	14.412
2	0.204	4.325	8.795	0.304	5.679	14.551
3	0.206	4.350	8.890	0.306	5.709	14.692
4	0.208	4.376	8.985	0.308	5.738	14.833
5	0.210	4.401	9.081	0.310	5.768	14.976
6	0.212	4.427	9.178	0.312	5.798	15.120
7	0.214	4.453	9.275	0.314	5.828	15.265
8	0.216	4.479	9.373	0.316	5.858	15.411
9	0.218	4.504	9.472	0.318	5.888	15.559
10	0.220	4.530	9.572	0.320	5.918	15.707
11	0.222	4.556	9.672	0.322	5.949	15.857
12	0.224	4.582	9.773	0.324	5.979	16.009
13	0.226	4.608	9.875	0.326	6.010	16.161
14	0.228	4.634	9.978	0.328	6.041	16.315
15	0.230	4.660	10.082	0.330	6.072	16.471
16	0.232	4.686	10.186	0.332	6.103	16.627
17	0.234	4.713	10.292	0.334	6.135	16.785
18	0.236	4.739	10.398	0.336	6.166	16.944
19	0.238	4.765	10.504	0.338	6.198	17.105
20	0.240	4.792	10.612	0.340	6.230	17.267
21	0.242	4.818	10.721	0.342	6.262	17.431
22	0.244	4.845	10.830	0.344	6.294	17.596
23	0.246	4.871	10.940	0.346	6.327	17.762
24	0.248	4.898	11.052	0.348	6.359	17.930
25	0.250	4.925	11.164	0.350	6.392	18.100
26	0.252	4.951	11.276	0.352	6.425	18.270
27	0.254	4.978	11.390	0.354	6.458	18.443
28	0.256	5.005	11.505	0.356	6.491	18.617
29	0.258	5.032	11.620	0.358	6.525	18.792
30	0.260	5.059	11.737	0.360	6.558	18.970
31	0.262	5.087	11.854	0.362	6.592	19.148
32	0.264	5.114	11.973	0.364	6.626	19.329
33	0.266	5.141	12.092	0.366	6.661	19.511
34	0.268	5.169	12.212	0.368	6.695	19.694
35	0.270	5.196	12.334	0.370	6.730	19.880
36	0.272	5.224	12.456	0.372	6.765	20.067
37	0.274	5.252	12.579	0.374	6.800	20.255
38	0.276	5.279	12.703	0.376	6.835	20.446
39	0.278	5.307	12.828	0.378	6.871	20.638
40	0.280	5.335	12.954	0.380	6.907	20.832
41	0.282	5.363	13.082	0.382	6.943	21.028
42	0.284	5.392	13.210	0.384	6.979	21.226
43	0.286	5.420	13.339	0.386	7.016	21.426
44	0.288	5.448	13.469	0.388	7.053	21.627
45	0.290	5.477	13.601	0.390	7.090	21.831
46	0.292	5.505	13.733	0.392	7.127	22.036
47	0.294	5.534	13.867	0.394	7.164	22.244
48	0.296	5.563	14.001	0.396	7.202	22.453
49	0.298	5.592	14.137	0.398	7.240	22.665
50	0.300	5.621	14.274	0.400	7.279	22.878
51						
52						
53						
54						

	a/W	KB\W/P	BEv/P	a/W	KB\W/P	BEv/P
57						
58	0.402	7.317	23.094	0.502	9.719	37.352
59	0.404	7.356	23.311	0.504	9.779	37.728
60	0.406	7.395	23.531	0.506	9.840	38.109
61	0.408	7.435	23.753	0.508	9.902	38.495
62	0.410	7.475	23.977	0.510	9.964	38.885
63	0.412	7.515	24.204	0.512	10.027	39.281
64	0.414	7.555	24.433	0.514	10.091	39.681
65	0.416	7.596	24.664	0.516	10.155	40.086
66	0.418	7.637	24.897	0.518	10.220	40.497
67	0.420	7.678	25.133	0.520	10.286	40.913
68	0.422	7.720	25.371	0.522	10.352	41.334
69	0.424	7.762	25.611	0.524	10.419	41.761
70	0.426	7.804	25.854	0.526	10.487	42.193
71	0.428	7.847	26.100	0.528	10.556	42.631
72	0.430	7.890	26.348	0.530	10.625	43.075
73	0.432	7.933	26.598	0.532	10.695	43.524
74	0.434	7.977	26.852	0.534	10.766	43.980
75	0.436	8.021	27.108	0.536	10.838	44.441
76	0.438	8.065	27.366	0.538	10.911	44.909
77	0.440	8.110	27.628	0.540	10.984	45.383
78	0.442	8.155	27.892	0.542	11.058	45.864
79	0.444	8.201	28.159	0.544	11.134	46.351
80	0.446	8.247	28.429	0.546	11.210	46.845
81	0.448	8.293	28.701	0.548	11.286	47.346
82	0.450	8.340	28.977	0.550	11.364	47.853
83	0.452	8.387	29.256	0.552	11.443	48.368
84	0.454	8.434	29.537	0.554	11.523	48.890
85	0.456	8.482	29.822	0.556	11.603	49.419
86	0.458	8.531	30.110	0.558	11.685	49.956
87	0.460	8.579	30.401	0.560	11.767	50.501
88	0.462	8.629	30.696	0.562	11.851	51.053
89	0.464	8.678	30.993	0.564	11.935	51.613
90	0.466	8.729	31.294	0.566	12.021	52.182
91	0.468	8.779	31.599	0.568	12.108	52.758
92	0.470	8.830	31.907	0.570	12.195	53.343
93	0.472	8.882	32.218	0.572	12.284	53.937
94	0.474	8.934	32.533	0.574	12.374	54.539
95	0.476	8.987	32.852	0.576	12.465	55.151
96	0.478	9.040	33.174	0.578	12.558	55.771
97	0.480	9.093	33.500	0.580	12.651	56.401
98	0.482	9.147	33.830	0.582	12.745	57.041
99	0.484	9.202	34.164	0.584	12.841	57.690
100	0.486	9.257	34.501	0.586	12.938	58.349
101	0.488	9.313	34.843	0.588	13.037	59.018
102	0.490	9.369	35.189	0.590	13.136	59.698
103	0.492	9.426	35.538	0.592	13.237	60.388
104	0.494	9.483	35.892	0.594	13.339	61.089
105	0.496	9.541	36.251	0.596	13.443	61.801
106	0.498	9.600	36.613	0.598	13.548	62.524
107	0.500	9.659	36.981	0.600	13.654	63.259
108						
109						
110						
111						
112						

	a/W	KB√W/P	BEv/P	a/W	KB√W/P	BEv/P
113	0.802	13.762	64.005	0.702	21.784	124.562
114	0.804	13.871	64.763	0.704	22.019	126.484
115	0.806	13.982	65.534	0.706	22.260	128.448
116	0.808	14.094	66.317	0.708	22.504	130.455
117	0.810	14.208	67.114	0.710	22.753	132.508
118	0.812	14.323	67.923	0.712	23.006	134.606
119	0.814	14.440	68.745	0.714	23.264	136.752
120	0.816	14.559	69.582	0.716	23.527	138.946
121	0.818	14.679	70.432	0.718	23.795	141.191
122	0.820	14.801	71.296	0.720	24.067	143.487
123	0.822	14.924	72.175	0.722	24.345	145.837
124	0.824	15.050	73.070	0.724	24.628	148.241
125	0.826	15.177	73.979	0.726	24.917	150.702
126	0.828	15.306	74.904	0.728	25.211	153.222
127	0.830	15.437	75.845	0.730	25.511	155.801
128	0.832	15.570	76.803	0.732	25.816	158.443
129	0.834	15.704	77.777	0.734	26.128	161.149
130	0.836	15.841	78.769	0.736	26.445	163.921
131	0.838	15.980	79.778	0.738	26.769	166.760
132	0.840	16.121	80.805	0.740	27.100	169.671
133	0.842	16.264	81.850	0.742	27.437	172.653
134	0.844	16.409	82.915	0.744	27.781	175.711
135	0.846	16.556	83.999	0.746	28.132	178.847
136	0.848	16.705	85.102	0.748	28.490	182.062
137	0.850	16.857	86.226	0.750	28.856	185.360
138	0.852	17.011	87.370	0.752	29.229	188.744
139	0.854	17.167	88.536	0.754	29.610	192.216
140	0.856	17.326	89.724	0.756	30.000	195.780
141	0.858	17.487	90.934	0.758	30.397	199.438
142	0.860	17.651	92.167	0.760	30.803	203.194
143	0.862	17.818	93.423	0.762	31.218	207.051
144	0.864	17.987	94.704	0.764	31.642	211.014
145	0.866	18.158	96.009	0.766	32.076	215.085
146	0.868	18.333	97.340	0.768	32.519	219.269
147	0.870	18.510	98.697	0.770	32.972	223.569
148	0.872	18.690	100.080	0.772	33.435	227.990
149	0.874	18.873	101.491	0.774	33.909	232.537
150	0.876	19.059	102.929	0.776	34.393	237.214
151	0.878	19.248	104.397	0.778	34.889	242.025
152	0.880	19.441	105.894	0.780	35.397	246.977
153	0.882	19.636	107.422	0.782	35.916	252.074
154	0.884	19.835	108.980	0.784	36.448	257.322
155	0.886	20.037	110.571	0.786	36.993	262.727
156	0.888	20.242	112.195	0.788	37.550	268.295
157	0.890	20.451	113.852	0.790	38.122	274.033
158	0.892	20.664	115.545	0.792	38.707	279.946
159	0.894	20.880	117.272	0.794	39.307	286.043
160	0.896	21.100	119.037	0.796	39.922	292.331
161	0.898	21.324	120.839	0.798	40.553	298.817
162	0.700	21.552	122.681	0.800	41.200	305.510
163						
164						
165						
166						
167						
168						

	a/W	KB <sup>2</sup> /W/P	BEv/P	a/W	KB <sup>2</sup> /W/P	BEv/P
169						
170	0.802	41.863	312.418	0.902	125.782	1445.692
171	0.804	42.544	319.551	0.904	129.842	1510.685
172	0.806	43.243	326.919	0.906	134.120	1579.983
173	0.808	43.960	334.530	0.908	138.631	1653.969
174	0.810	44.697	342.397	0.910	143.396	1733.070
175	0.812	45.453	350.529	0.912	148.433	1817.762
176	0.814	46.230	358.940	0.914	153.765	1908.577
177	0.816	47.029	367.641	0.916	159.416	2006.114
178	0.818	47.850	376.646	0.918	165.415	2111.042
179	0.820	48.695	385.968	0.920	171.792	2224.118
180	0.822	49.563	395.623	0.922	178.581	2346.199
181	0.824	50.457	405.625	0.924	185.820	2478.255
182	0.826	51.377	415.993	0.926	193.552	2621.389
183	0.828	52.324	426.742	0.928	201.825	2776.863
184	0.830	53.299	437.891	0.930	210.694	2946.121
185	0.832	54.303	449.461	0.932	220.220	3130.824
186	0.834	55.339	461.472	0.934	230.474	3332.890
187	0.836	56.406	473.946	0.936	241.536	3554.542
188	0.838	57.507	486.907	0.938	253.498	3798.372
189	0.840	58.643	500.380	0.940	266.466	4067.409
190	0.842	59.815	514.392	0.942	280.561	4365.221
191	0.844	61.025	528.970	0.944	295.928	4696.022
192	0.846	62.276	544.146	0.946	312.731	5064.830
193	0.848	63.567	559.951	0.948	331.165	5477.645
194	0.850	64.903	576.421	0.950	351.463	5941.701
195	0.852	66.284	593.592	0.952	373.897	6465.773
196	0.854	67.713	611.504	0.954	398.798	7060.595
197	0.856	69.192	630.198	0.956	426.561	7739.402
198	0.858	70.724	649.721	0.958	457.669	8518.668
199	0.860	72.311	670.121	0.960	492.715	9419.101
200	0.862	73.956	691.451	0.962	532.433	10467.018
201	0.864	75.662	713.765	0.964	577.746	11696.269
202	0.866	77.433	737.126	0.966	629.821	13150.980
203	0.868	79.271	761.597	0.968	690.159	14889.528
204	0.870	81.181	787.249	0.970	760.722	16990.438
205	0.872	83.166	814.158	0.972	844.110	19561.329
206	0.874	85.231	842.406	0.974	943.839	22752.870
207	0.876	87.380	872.081	0.976	1064.774	26781.188
208	0.878	89.617	903.279	0.978	1213.809	31965.194
209	0.880	91.949	936.105	0.980	1401.017	38791.264
210	0.882	94.380	970.671	0.982	1641.638	48030.918
211	0.884	96.917	1007.101	0.984	1959.736	60967.527
212	0.886	99.566	1045.529	0.986	2395.358	79864.855
213	0.888	102.335	1086.100	0.988	3019.735	109024.226
214	0.890	105.230	1128.976	0.990	3971.114	157456.154
215	0.892	108.260	1174.329	0.992	5551.905	246748.271
216	0.894	111.435	1222.354	0.994	8550.822	439952.992
217	0.896	114.763	1273.259	0.996	15714.312	992804.222
218	0.898	118.256	1327.276	0.998	44461.463	3982891.495

	a/W	KB $\sqrt{W/P}$	a/W	KB $\sqrt{W/P}$
1	0.010	0.089	0.510	0.757
2	0.020	0.125	0.520	0.771
3	0.030	0.154	0.530	0.785
4	0.040	0.177	0.540	0.799
5	0.050	0.198	0.550	0.814
6	0.060	0.218	0.560	0.829
7	0.070	0.235	0.570	0.845
8	0.080	0.252	0.580	0.861
9	0.090	0.267	0.590	0.877
10	0.100	0.282	0.600	0.894
11	0.110	0.296	0.610	0.912
12	0.120	0.310	0.620	0.930
13	0.130	0.323	0.630	0.949
14	0.140	0.335	0.640	0.968
15	0.150	0.348	0.650	0.989
16	0.160	0.360	0.660	1.010
17	0.170	0.372	0.670	1.031
18	0.180	0.383	0.680	1.054
19	0.190	0.395	0.690	1.078
20	0.200	0.406	0.700	1.103
21	0.210	0.417	0.710	1.129
22	0.220	0.428	0.720	1.156
23	0.230	0.439	0.730	1.185
24	0.240	0.450	0.740	1.215
25	0.250	0.460	0.750	1.247
26	0.260	0.471	0.760	1.280
27	0.270	0.482	0.770	1.316
28	0.280	0.492	0.780	1.354
29	0.290	0.503	0.790	1.395
30	0.300	0.513	0.800	1.438
31	0.310	0.524	0.810	1.485
32	0.320	0.535	0.820	1.535
33	0.330	0.545	0.830	1.589
34	0.340	0.556	0.840	1.649
35	0.350	0.567	0.850	1.714
36	0.360	0.577	0.860	1.785
37	0.370	0.588	0.870	1.864
38	0.380	0.599	0.880	1.952
39	0.390	0.610	0.890	2.052
40	0.400	0.622	0.900	2.166
41	0.410	0.633	0.910	2.298
42	0.420	0.644	0.920	2.454
43	0.430	0.656	0.930	2.640
44	0.440	0.668	0.940	2.870
45	0.450	0.680	0.950	3.165
46	0.460	0.692	0.960	3.562
47	0.470	0.705	0.970	4.141
48	0.480	0.717	0.980	5.106
49	0.490	0.730	0.990	7.270
50	0.500	0.743		

	K cal SE(T)			
	a/W	KB $\sqrt$ W/P	a/W	KB $\sqrt$ W/P
1	0.010	0.199	0.510	3.696
2	0.020	0.283	0.520	3.858
3	0.030	0.348	0.530	4.029
4	0.040	0.404	0.540	4.210
5	0.050	0.455	0.550	4.402
6	0.060	0.501	0.560	4.605
7	0.070	0.546	0.570	4.820
8	0.080	0.588	0.580	5.049
9	0.090	0.630	0.590	5.292
10	0.100	0.670	0.600	5.551
11	0.110	0.710	0.610	5.828
12	0.120	0.750	0.620	6.123
13	0.130	0.790	0.630	6.440
14	0.140	0.830	0.640	6.779
15	0.150	0.871	0.650	7.144
16	0.160	0.912	0.660	7.537
17	0.170	0.953	0.670	7.960
18	0.180	0.996	0.680	8.418
19	0.190	1.039	0.690	8.915
20	0.200	1.083	0.700	9.455
21	0.210	1.129	0.710	10.043
22	0.220	1.175	0.720	10.685
23	0.230	1.224	0.730	11.389
24	0.240	1.273	0.740	12.163
25	0.250	1.324	0.750	13.018
26	0.260	1.377	0.760	13.965
27	0.270	1.431	0.770	15.019
28	0.280	1.488	0.780	16.196
29	0.290	1.546	0.790	17.519
30	0.300	1.607	0.800	19.013
31	0.310	1.670	0.810	20.709
32	0.320	1.735	0.820	22.650
33	0.330	1.803	0.830	24.886
34	0.340	1.873	0.840	27.483
35	0.350	1.947	0.850	30.527
36	0.360	2.023	0.860	34.133
37	0.370	2.103	0.870	38.457
38	0.380	2.186	0.880	43.713
39	0.390	2.273	0.890	50.206
40	0.400	2.363	0.900	58.381
41	0.410	2.458	0.910	68.915
42	0.420	2.557	0.920	82.875
43	0.430	2.660	0.930	102.038
44	0.440	2.769	0.940	129.571
45	0.450	2.882	0.950	171.626
46	0.460	3.002	0.960	241.673
47	0.470	3.127	0.970	374.883
48	0.480	3.258	0.980	693.876
49	0.490	3.397	0.990	1977.438
50	0.500	3.543		

# Relationship of K Solutions

$$K = \sigma \sqrt{\pi a} F\left(\frac{a}{W}\right)$$

$$\sigma = \frac{P}{BW};$$

$$K = \frac{P}{BW} \sqrt{\pi a} F = \frac{P}{BW^{1/2}} \sqrt{\frac{\pi a}{W}} F = \frac{Pf}{BW^{1/2}};$$

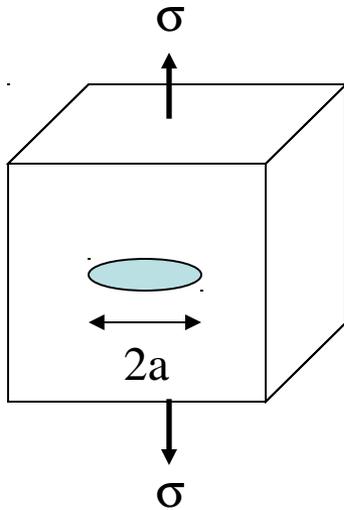
$$f = \sqrt{\frac{\pi a}{W}} F$$

# Embedded and surface flaws

- Most K solutions are for straight fronted cracks, 2-D
- Many cracks in structures are embedded or surface 3-D cracks

# 3-dimensional cracks

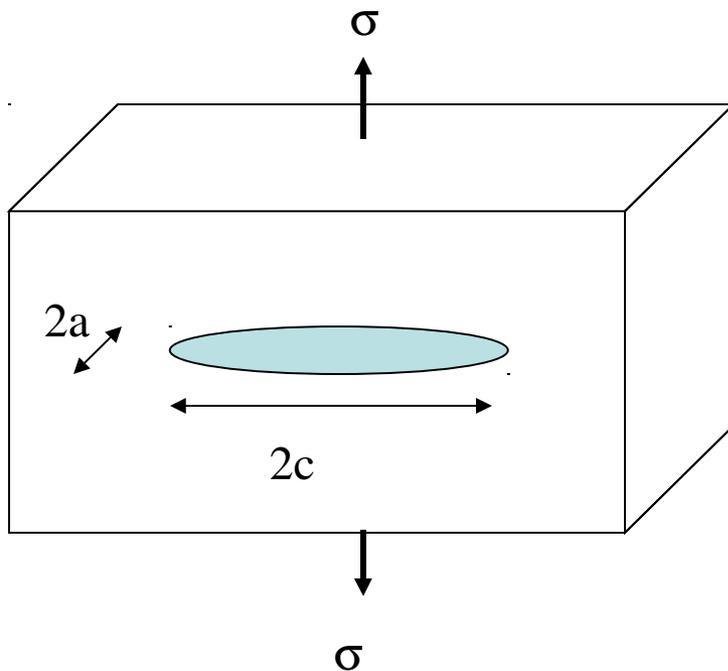
Volume with imbedded penny shaped crack (circular);  
radius,  $a$ ; stressed by  $\sigma$



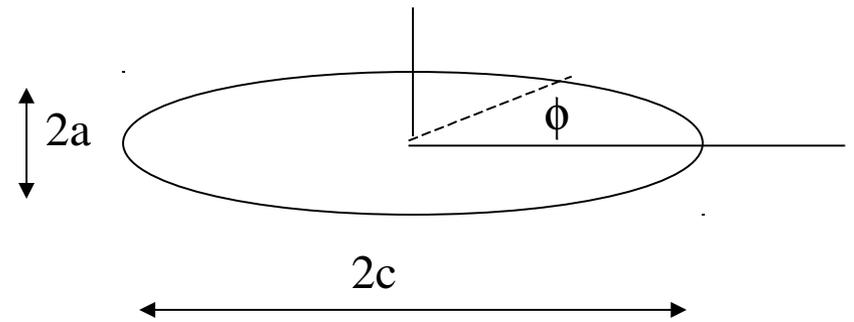
$$K = \frac{2}{\pi} \sigma \sqrt{\pi a}$$

# Elliptical flaw

- The general internal flaw is elliptical,  $2c$  by  $2a$



$$K = \sigma \sqrt{\frac{\pi a}{Q}} \left[ \sin^2 \phi + \left( \frac{a}{c} \right)^2 \cos^2 \phi \right]^{1/4}$$



## Surface Flaw K Expression

$$K = 1.122\sigma \sqrt{\frac{\pi a}{Q}} \left[ \sin^2 \phi + \left(\frac{a}{c}\right)^2 \cos^2 \phi \right]^{1/4}$$

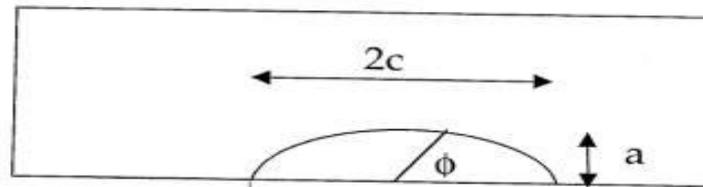
Flaw is semi-elliptical:

Depth = a

Surface length = 2c

$\phi$  = angle from surface

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65}$$



# Example

- Calculate K for a CT (table):

$$W = 2 \text{ in}, B = 1 \text{ in}, a = 1.2 \text{ in}, P = 8 \text{ kips}$$

- $a/W = 1.2/2 = 0.6$ ,  $f = 13.65$

$$K = \frac{Pf}{B\sqrt{W}} = \frac{8 * 13.65}{1\sqrt{2}} = 77 \text{ ksi}\sqrt{\text{in}}$$

# Example 2

- CCT (Picture 2.1)
- $2b = 10$  in,  $2a = 2$  in,  $\sigma = 50$  ksi
- $a/b = 1/5 = 0.2$ ;  $F = 1.025$

$$K = \sigma \sqrt{\pi a} F = 50 \sqrt{\pi (1.0)} 1.025 = 90.8 \text{ ksi} \sqrt{\text{in}}$$