

# A Not-So-Random View of Randomness

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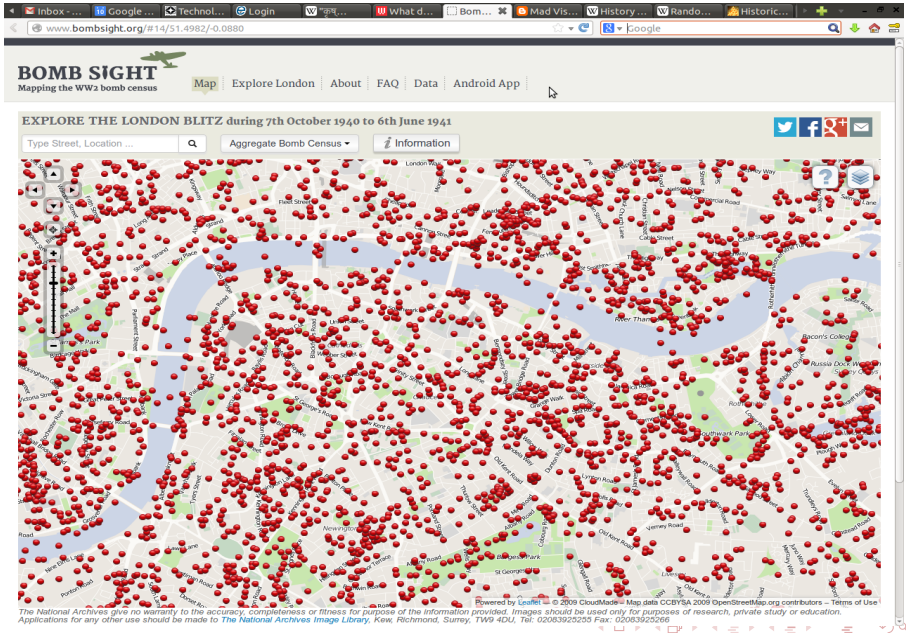
# Example 1.1: Placement of stars in the sky



## Example 1.1: Glowworms in a New Zealand Cave



# Example 1.1: London Bombing, 1941



## Example 1.2: Games of chance



*Axis Theory Photos*

## Example 1.2: Gambling, Ancient Rome




## Example 1.2: Gambling, Ancient India



# Example 1.2: Lottery

**STATE LOTTERY 1761,**



**The S C H E M E.**

No of Prizes.	Value of each.	Total Value.
2	of 10000	is 20000
2	5000	10000
4	2000	8000
10	1000	10000
20	500	10000
100	100	10000
201	50	10050
11606	20	232120
<b>11945 Prizes</b>		<b>310170</b>
	First Drawn	500
	Last Drawn	1000
<b>48055 Blanks, at 6l. each</b>		<b>288330</b>

60000 Tickets, at 10 l. each 600000  
 N. B. Begins drawing *Monday, November 16th.*  
 The TICKETS, SHARES and CHANCES of Tickets as usual,  
 Sold and Registered by

## A and C. CORBETT,

Bookfellers and Publishers,  
 At their *Correct State Lottery-Office*, kept many Years past  
 at *Addison's-Head*, facing *St. Dunstan's Church, Fleet-street*,  
 Who have in former Lotteries Sold, Shared, and Registered no less than  
 EIGHTY-EIGHT of the Capital Prizes from 5000. to 10000. each.  
 Letters (Post paid) duly answered, and Schemes given gratis.  
 And the most Worthy gives for Prizes in the Last, and Blanks and Prizes in  
 former Lotteries. Only Four Blanks to a Prize.



**दिवाली भव्यतम लॉटरी**

महाराष्ट्र राज्य लॉटरी  
 DRAW NO. - 3  
 DRAW DATE  
 15-11-2013

लॉटरी भंडार

लॉटरी भंडार

लाल इमली गली, जुन्नवारी, नागपूर. 2.  
 0712-2761105 फॅक्स. 0712-2763047.

मालिका	DL-01	DL-02	DL-03	DL-04	DL-05	DL-06	DL-07	DL-08	DL-09	DL-10
पहिले बहिये अ. 2 करोड	<b>सब स्टॉ. मे. अंबिका एजन्सी (दादर-मुंबई)</b>								<b>87697</b>	<b>स्टॉ. मे. प्रिंस एजन्सी (मुंबई)</b>
दुसरे बहिये अ. 25 लाख	<b>73518</b>								<b>स्टॉ. मे. प्रिंस एजन्सी (मुंबई)</b>	<b>सब स्टॉ. मे. रुक्मिणी एंटरप्रायझेस (कोल्हापूर)</b>
तिसरे बहिये अ. 11 लाख	<b>97049</b>				<b>स्टॉ. मे. प्रिंस एजन्सी (मुंबई)</b>			<b>सब स्टॉ. मे. श्री गणेश लॉटरी एजन्सी (दादर-मुंबई)</b>		
चौथे बहिये अ. 2 लाख	<b>स्टॉ. मे. प्रिंस एजन्सी (मुंबई)</b>				<b>97941</b>			<b>सब स्टॉ. मे. त्रिमुर्ती लॉटरी एजन्सी (मिरजोड-मुंबई)</b>		
पाचवे बहिये अ. 1 लाख	01068 ३. अंबिका एजन्सी (दादर-मुंबई)	62476 I. B. NAGPUR	09548 ३. श्री गिरीश सर्व (दादर) (दादर-मुंबई)	31211 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)	67208 ३. विठ्ठल सर्व (दादर) (दादर-मुंबई)	44516 ३. वरदा सर्व (दादर) (दादर-मुंबई)	83221 ३. वरदा सर्व (दादर) (दादर-मुंबई)	00176 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)	38352 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)	32281 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)
	84198 ३. वरदा सर्व (दादर) (दादर-मुंबई)	62615 I. B. NAGPUR	31734 ३. वरदा सर्व (दादर) (दादर-मुंबई)	33340 ३. वरदा सर्व (दादर) (दादर-मुंबई)	78442 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)	60271 ३. श्री मंगेश सर्व (दादर) (दादर-मुंबई)	35200 I. B. NAGPUR	73038 ३. अरबास सर्व (दादर) (दादर-मुंबई)	39524 ३. वरदा सर्व (दादर) (दादर-मुंबई)	69457 ३. वरदा सर्व (दादर) (दादर-मुंबई)
	96864 ३. वरदा सर्व (दादर) (दादर-मुंबई)	92835 ३. श्री गिरीश सर्व (दादर) (दादर-मुंबई)	71637 I. B. NAGPUR	90881 I. B. NAGPUR	82275 ३. अंबिका एंटरप्रायझेस (दादर) (दादर-मुंबई)	94734 ३. वरदा सर्व (दादर) (दादर-मुंबई)	52995 I. B. NAGPUR	75549 ३. वरदा सर्व (दादर) (दादर-मुंबई)	89964 ३. अरबास सर्व (दादर) (दादर-मुंबई)	70251 ३. अरबास सर्व (दादर) (दादर-मुंबई)
सहावे बहिये अ. 7000/-	9968	4156	2023	7302	0310	5576	6268	7973	3647	6360
सातवे बहिये अ. 5000/-	0063	1768	5983	0919	5660	1699	5757	0125	1276	8831
आठवे बहिये अ. 5000/-	1510	0713	2120	5187	9330	0162	8282	7582	7653	2518

सर्व मालिकासाठी • COMMON TO ALL SERIES • सर्व मालिकासाठी • COMMON TO ALL SERIES

आठवे बहिये अ. 1000/- (सर्व मालिकासाठी)										
1299	1302	2828	4493	5922	6081	6123	6713	6903	8053	
दहावे बहिये अ. 700/- (सर्व मालिकासाठी)										
1061	1143	1201	2173	3662	4115	4528	5358	6448	9578	
अकरावे बहिये अ. 500/- (सर्व मालिकासाठी)										
1710	3718	5500	6027	8559	8689	9413	9609	9680	9798	
बारावे बहिये अ. 200/- (सर्व मालिकासाठी)										

## 9599

या क्रमांकावर तसेच या क्रमांकाच्या पुढील तसेच मागील 10 या क्रमांकांच्या निकीटार तसेच 200/- चे बहिये सर्व मालिकांसाठी देणेत येईल.

### बाराव्या क्रमांकाच्या बक्षिसाचे सर्व क्रमांक मागे पहा.

5<sup>th</sup> PRIZE OF RS. 1 LAKH SOLD BY STOCKIST :- LOTTERY BHANDAR, NAGPUR, (LUCKY LIFAFI DHAMAKA)  
 (DL-02-82618) AGENT :- OM SAI LOTTERY RAGHULI NAGAR.  
 (DL-02-84348) AGENT :- THAKUR LOTTERY.  
 (DL-04-90818) AGENT :- LUCKY LIFAFI  
 (DL-07-35200) AGENT :- JAI DURGA LOTTERY CENTER.  
 (DL-07-52989) AGENT :- PALARAM LOTTERY CENTER.

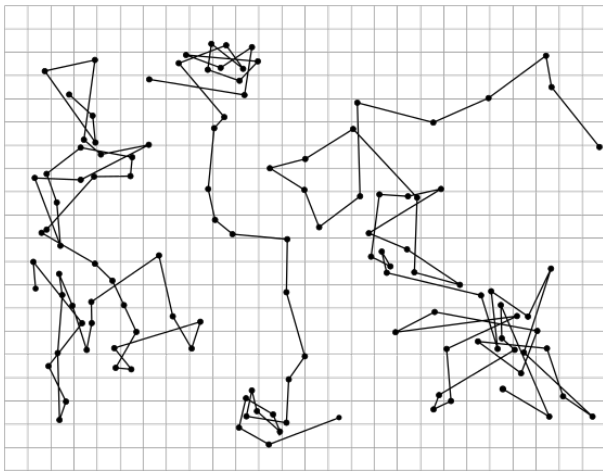


## Example 1.2: Lottery



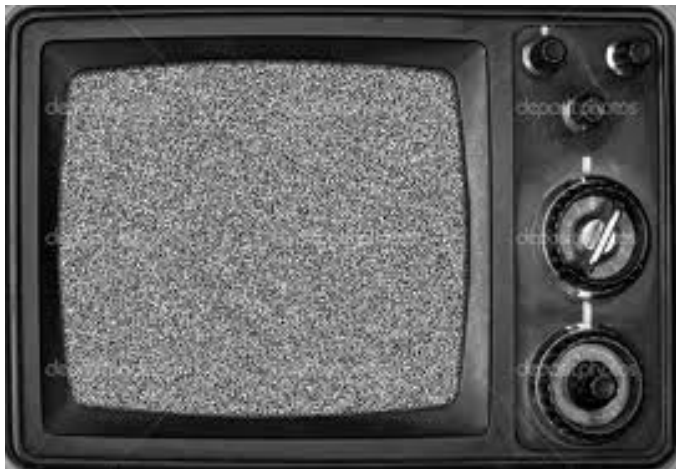
"The State Lottery" by Vincent van Gogh, 1882

## Example 1.3: Brownian Motion



Three tracings of the motion of colloidal particles of radius  $0.53 \mu\text{m}$ , as seen under the microscope. Successive positions every 30 seconds are joined by straight line segments. Mesh size:  $3.2 \mu\text{m}$ . (Jean Baptiste Perrin, *Mouvement brownien et réalité moléculaire*, Ann. de Chimie et de Physique (VIII) 18, 5-114, 1909).

## Example 1.3: Noise in Electrical Circuits



## Example 1.3: Pune Traffic, 20xx

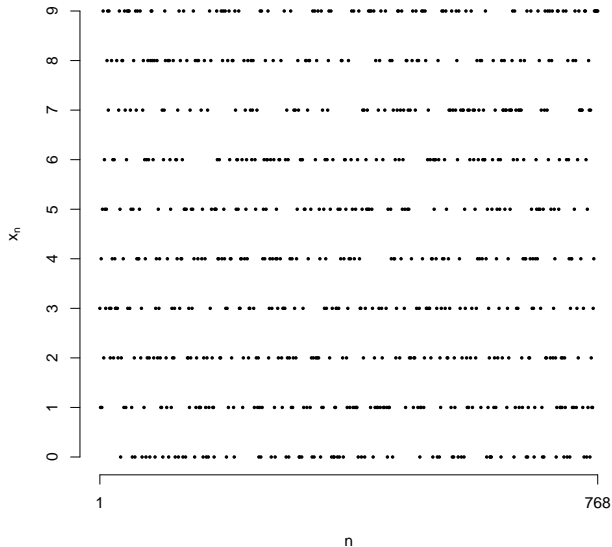


## Randomness

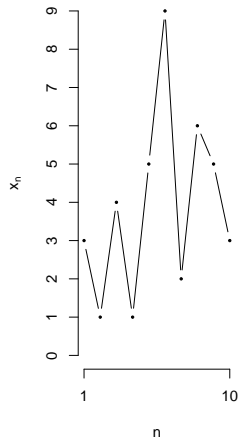
This supposedly “simple” and “intuitive” notion can be notoriously intriguing ...



# Example 2: Is this sequence of decimal digits random?

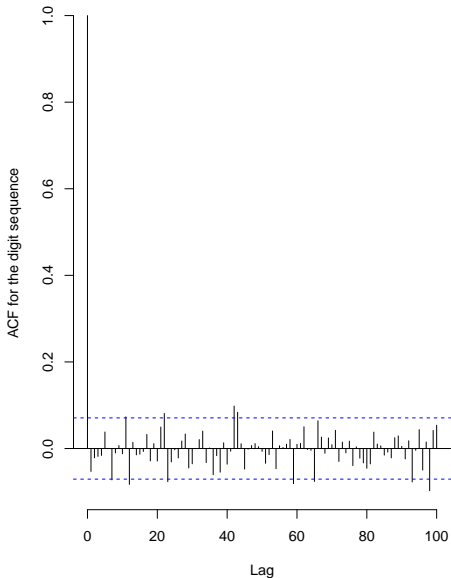
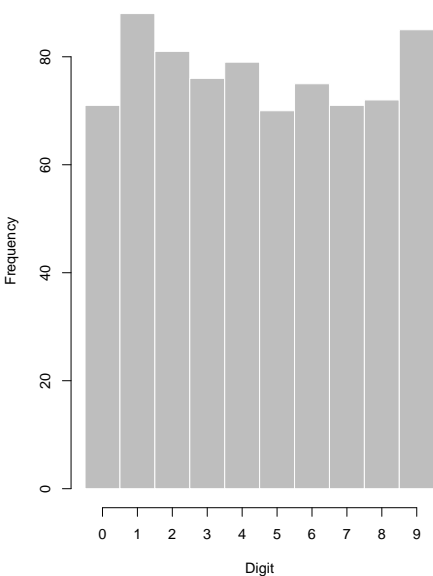


Initial portion



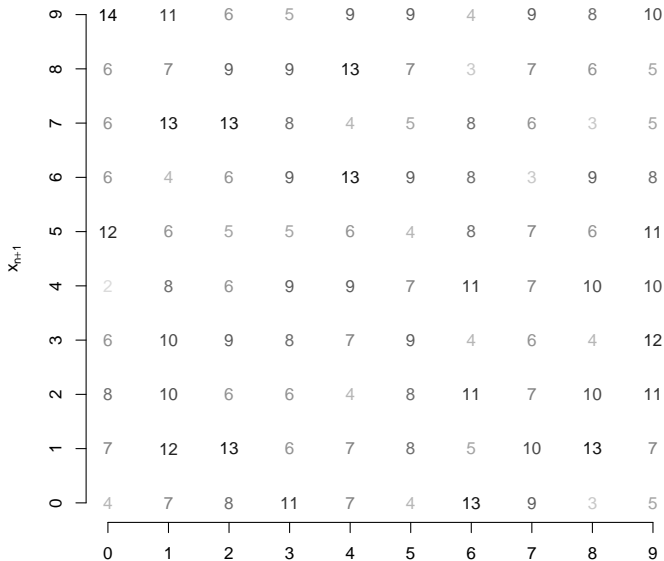
# Example 2: Is this sequence of decimal digits random?

Statistical indications of "randomness"?



# Example 2: Is this sequence of decimal digits random?

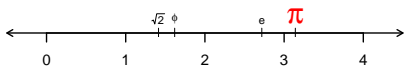
Co-occurrence counts in successive pairs





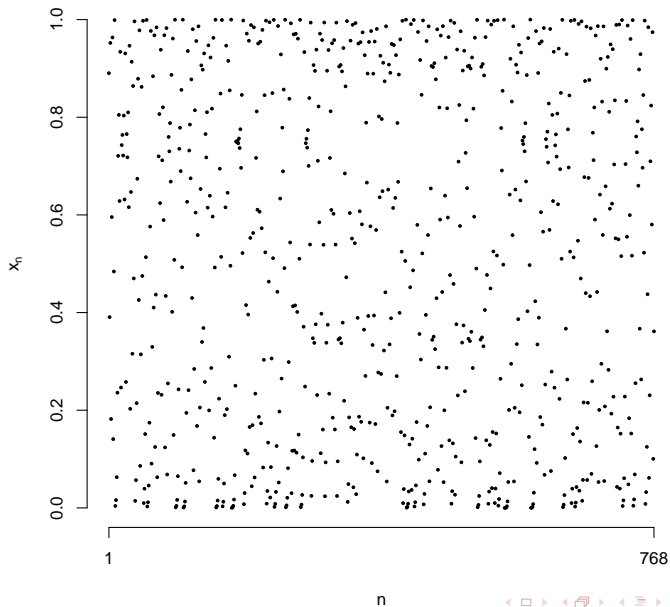
## Example 2: Is this sequence of decimal digits random?

3.14159265358979323846264338327950288419716939937510582097  
494459230781640628620899862803482534211706798214808651328  
230664709384460955058223172535940812848111745028410270193  
852110555964462294895493038196442881097566593344612847564  
823378678316527120190914564856692346034861045432664821339  
360726024914127372458700660631558817488152092096282925409  
171536436789259036001133053054882046652138414695194151160  
943305727036575959195309218611738193261179310511854807446  
237996274956735188575272489122793818301194912983367336244  
065664308602139494639522473719070217986094370277053921717  
629317675238467481846766940513200056812714526356082778577  
134275778960917363717872146844090122495343014654958537105  
079227968925892354201995611212902196086403441815981362977  
477130996051870721134999999... Sequence of decimal digits of  $\pi$ !



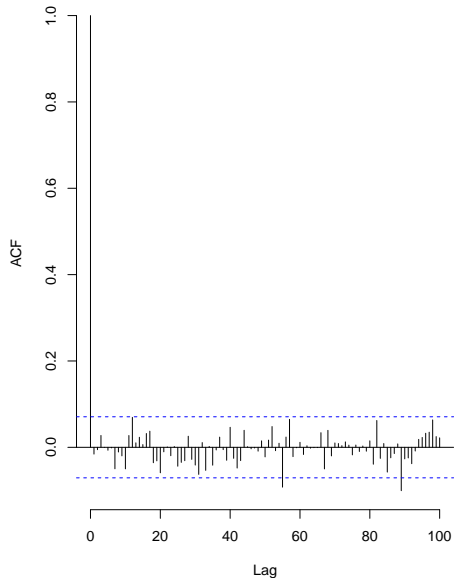
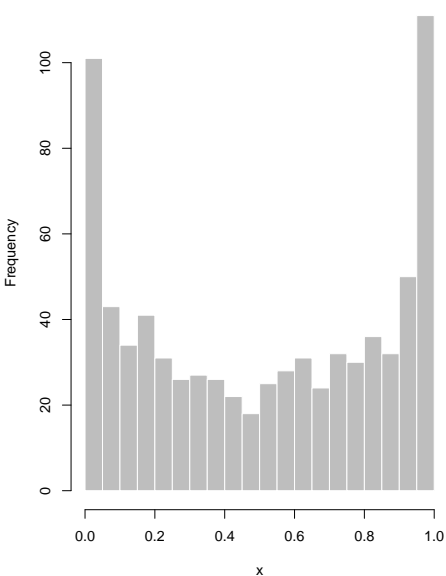
A perfectly well-defined number!

## Example 3: Is this signal random?

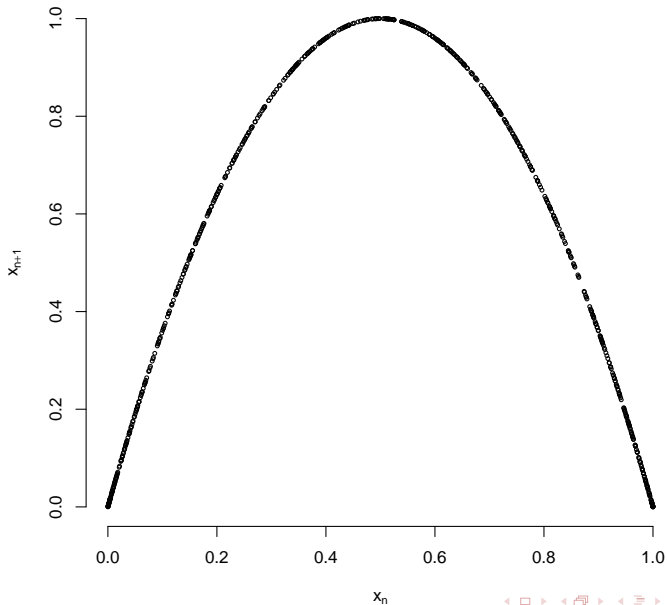


# Example 3: Is this signal random?

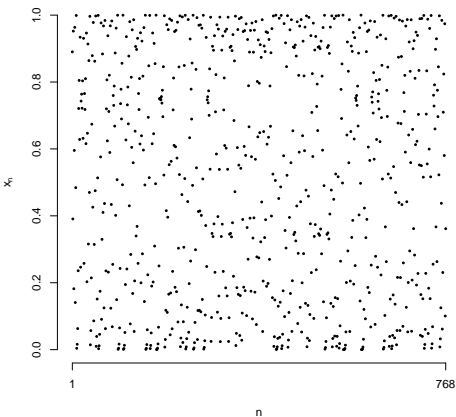
Statistical indications of "randomness"?



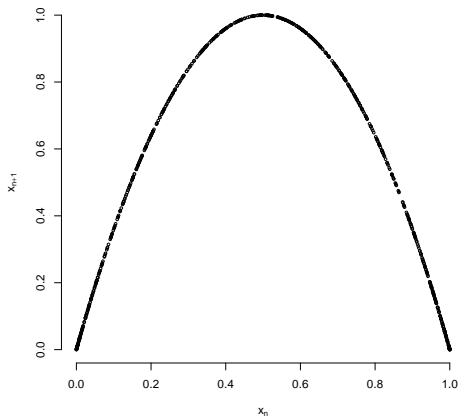
## Example 3: Is this signal random?



## Example 3: Is this signal random?



chaotic, unpredictable, patternless, ...



... but a delay plot reveals the rule

The logistic map: Iterate  $X_{n+1} = 4X_n(1 - X_n)$  ( $0 \leq X_0 \leq 1$ )

# Connotations of *randomness*

A sequence of numbers which

- lacks predictability
- lacks (easily) recognizable pattern or rule
- contains information which cannot be compressed into an equivalent but shorter representation
- is indistinguishable from realizations of a truly random process
- has statistical independence (in some contexts)
- ...

*Imprecise as definition,  
uses circular reasoning,*

...

*(but you know what is meant)*

# An operational definition of randomness

Mais quand une regle est fort composée, ce qui luy est conforme, passe pour irrégulier.

But when a rule is extremely complex, that which conforms to it passes for random.

Gottfried Wilhelm 

*Discourse on Metaphysics (1686)*

as quoted at <http://en.wikiquote.org/wiki/Randomness>

# Generating “random” numbers

We may, therefore, use **deterministic** means to generate a sequence of numbers that **appear** random ...